PROCEEDINGS

American Society of Civil Engineers

OCTOBER

1948

AMERICAN SOCIETY OF CIVIL ENGINEERS FOUNDED 1852

VOLUME 74 NO. 8

AMERICAN SOCIETY OF CIVIL ENGINEERS

PROCEEDINGS OF THE SOCIETY

PROCEEDINGS

OF THE

AMERICAN SOCIETY OF CIVIL ENGINEERS

VOL. 74 OCTOBER, 1948 No. 8

TECHNICAL PAPERS

AND

DISCUSSIONS

A list of "Current Papers and Discussions" may be found on the page preceding the table of contents

The Society is not responsible for any statement made or opinion expressed in its publications.

Published monthly, except July and August, at Prince and Lemon Streets, Lancaster, Pa., by the American Society of Civil Engineers. Editorial and General Offices at 33 West Thirty-ninth Street, New York 18, N. Y. Reprints from this publication may be made on condition that the full title of paper, name of author, page reference, and date of publication by the Society are given.

Entered as Second-Class Matter, September 23, 1937, at the Post Office at Lancaster, Pa., under the Act of March 3, 1879. Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 5, 1918.

Subscription (if entered before January 1) $8.00 per annum Price $1.00 per copy

Printed in the United States of America

CURRENT PAPERS AND DISCUSSIONS

	Published	Discussion closes
Bergendorff, R. N., and Sorkin, Josef. Mississippi River Bridge at Dubuque, Iowa	June, 1947	
Discussion in Sept., Oct., 1947, May, 1948		Closed*
Symposium: Problems and Control of Decentralization in Urban Areas	Nov., 1947	
Discussion in Mar., June, 1948		Closed*
Moore, William W. Experiences with Predetermining Pile Lengths	Nov., 1947	
Discussion in Mar., June, 1948		Closed*
Turner, Robert E. Operation of the Conowingo Hydroelectric Plant	Nov., 1947	
Discussion in June, Sept., 1948		Closed*
Hickerson, T. F. Determination of Position and Azimuth by Simple and Accurate Methods	Nov., 1947	
Discussion in June, 1948		Closed*
Bell, S. J. A Centroidal Method of Rigid-Frame Analysis	Nov., 1947	
Discussion in June, Sept., 1948		Closed*
Loring, Samuel J. Experimental Determination of Vibration Characteristics of Structures	Dec., 1947	
Discussion in Mar., June, Sept., 1948		Closed*
Chen Pei-ping. Matrix Analysis of Pin-Connected Structures	Dec., 1947	
Discussion in June, Oct., 1948		Closed*
Andrew, Charles E. Unusual Design Problems—Second Tacoma Narrows Bridge	Dec., 1947	
Discussion in June, 1948		Closed*
Stewart, Ralph W. Analysis of Frames with Elastic Joints	Dec., 1947	
Discussion in May, June, Sept., 1948		Closed*
Levinton, Zusse. Elastic Foundations Analyzed by the Method of Redundant Reactions	Dec., 1947	
Discussion in Sept., Oct., 1948		Closed*
Harza, L. F. The Significance of Pore Pressure in Hydraulic Structures	Dec., 1947	
Discussion in June, Sept., Oct., 1948		Nov. 1, 1948
Symposium: Lateral Earth Pressures on Flexible Retaining Walls	Jan., 1948	
Discussion in June, Sept., Oct., 1948		Nov. 1, 1948
Neiman, A. S. Shearing Stress Distribution in Box Girders with Multiple Webs	Feb., 1948	
Discussion in June, Sept., 1948		Nov. 1, 1948
Symposium: Highway Bridge Floors	Mar., 1948	
Discussion in Sept., Oct., 1948		Nov. 1, 1948
DuVal, Miles P. The Marine Operating Problems, Panama Canal, and The Solution	Feb., 1947	Dec. 1, 1948
Claybourn, J. G. Sea Level Plan for Panama Canal	Feb., 1947	
Discussion in Feb., June, 1948		Dec. 1, 1948
Symposium: Panama Canal—The Sea-Level Project	Apr., 1948	
Discussion in June, Sept., Oct., 1948		Dec. 1, 1948
Chellis, Robert D. The Relationship Between Pile Formulas and Load Tests	May, 1948	
Discussion in Sept., 1948		Dec. 1, 1948
Wimer, H. Dudley, Jr. Stability of Thin-Walled Compression Members	May, 1948	Dec. 1, 1948
Scharff, Maurice R. Valuation and Depreciation of Public Utility Property	June, 1948	
Discussion in Oct., 1948		Jan. 1, 1948
Report. Review of Slope Protection Methods: Report of the Subcommittee on Slope Protection of the Committee on Earth Dams of the Soil Mechanics and Foundations Division	June, 1948	
Discussion in Oct., 1948		Jan. 1, 1948
Symposium: Industrial Wastes	Sept., 1948	Feb. 1, 1949
Chu, Chung-jui. Adjustment of the "Shoran" Triangulation	Sept., 1948	Feb. 1, 1949
Feil, Louis G. Unusual Design Problems, Harlan County Dam	Sept., 1948	Feb. 1, 1949
Anderson, A. A. Expansion Joint Practice in Highway Construction	Sept., 1948	Feb. 1, 1949
Tsien, Ling-hi. A Simplified Method of Analyzing Suspension Bridges	Sept., 1948	Feb. 1, 1949
Tsai, Fang-Yin. Flexural Constants of Haunched Beams by Area Computation	Sept., 1948	Feb. 1, 1949

NOTE.—The closing dates herein published are final except when names of prospective discussers are registered for special extension of time.

* Publication of closing discussion pending.

CONTENTS FOR OCTOBER, 1948

PAPER

REPORT

DISCUSSIONS

CONTENTS FOR OCTOBER, 1948 (*Continued*)

GUIDEPOST FOR TECHNICAL READERS
OCTOBER, 1948, *PROCEEDINGS*

Items of value or significance to readers in various fields are here listed. For convenience the distribution is given in terms of the Society's Technical Divisions

(A constant effort is made to supply technical material to Society members, over the entire range of possible interest. In so far as your specialty may be covered inadequately in the foregoing list, this fact is a gage of the need for your help toward improvement.—Ed.)

HINTS TO AUTHORS

Those who are planning papers for submission to PROCEEDINGS *will expedite Committee action measurably by first studying the standard instructions as to style, content, and format. For reprints, address the Manager, Technical Publications, 33 W. 39th St., New York 18, N. Y.*

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

DYNAMIC INSTABILITY OF TRUSS-STIFFENED SUSPENSION BRIDGES UNDER WIND ACTION

By Friedrich Bleich,[1] M. ASCE

Synopsis

An attempt is made in this paper to approach by rational analysis the problem of self-excitation of vibrations in suspension bridges. A mathematical solution of the problem of aerodynamic instability of truss-stiffened suspension bridges under wind action, which likewise applies to suspension bridges with extremely shallow stiffening girders, is presented herein. The theory leads to the explanation of the mechanism of catastrophic self-excitation in truss-type bridges, and ultimately to a rational method of bridge design against wind in order to assume aerodynamic stability.

1. Introduction

The tragic failure of the Tacoma Narrows Bridge in the State of Washington in the fall of 1940 directed the attention of the engineering profession to a problem only recently recognized as one of vital importance in suspension bridge design. The disaster brought the question of vibrations of suspension bridges under wind action into the focus of interest of bridge designers and investigators in the field of aerodynamics.

In structural engineering, wind forces customarily have been considered static forces—that is, steady forces acting on a structure at rest, like the horizontal wind forces usually taken into account in structural design. The heavy undulations of the Tacoma Narrows Bridge, which finally led to its collapse, demonstrated, however, that, even under action of a steady wind, unsteady forces (actually, periodic forces) may operate to excite oscillations with progressively augmented amplitudes, which may reach dangerous or even destructive proportions. These oscillations have been recognized as so-called self-induced or self-excited vibrations, closely connected with a peculiar state of dynamic instability of the vibrating structure.

Note. Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **March 1, 1949.**

[1] With Frankland and Lienhard, Cons. Engrs., New York, N. Y.; formerly Consultant to Advisory Board on the Investigation of Suspension Bridges, Public Roads Administration, Washington, D. C.

Elaborate research at the Daniel Guggenheim Aeronautical Laboratories at the California Institute of Technology in Pasadena, and at the University of Washington Laboratory in Seattle, instituted and directed by engineers of the Washington Toll Bridge Authority and Public Roads Administration and sponsored by these two organizations, has brought to light many significant facts and afforded much insight into the somewhat obscure mechanism of self-excitation in suspension bridges under wind action. The well-interpreted results of the wind tunnel tests on various models now provide the necessary fundamentals for an attempt to develop, with a prospect of success, a mathematical theory of dynamic instability under wind action, at least for a certain group of suspension bridges.

The purpose of a mathematical investigation into the problem of dynamic instability is a twofold one. The primary aim is the development of a theory with which observed facts can be correlated, which may be useful as a guide in conducting further laboratory tests, and which in turn may confirm the theory or indicate how the theory can be improved. The other purpose is to establish the theoretical and experimental foundation for a rational method of design permitting the specification of the condition of instability in any particular case.

The problem of aerodynamically excited oscillations in any suspension bridge structure under action of a steady uniform wind flow is one having a dual aspect. The problem proper is one of the mechanics of fluid flow involving the mutual relations between the shape of the suspended structure and the character and magnitude of the air forces acting on it. These relations and the interrelations between the air forces and the inherent dynamic properties of the structure form the basic feature of this phase of the research. The second aspect of the problem involves the detailed investigation of the response of the structure to the imposed aerodynamic forces—that is, the investigation of those conditions which operate to induce dynamic instability in any suspension bridge subjected to a steady wind stream.

The first-mentioned phase of the problem—the determination of the aerodynamic forces which act upon the oscillating structure—appears to be, in the first instance, a subject of laboratory research in view of the complexity of the geometric form and the variety of the structural make-up of the suspended floor. However, the possibility must not be overlooked that, at least in certain cases. perhaps an idealized enveloping surface of the floor structure may actually control the aerodynamic behavior of the structure, and that the influence that details of the geometric form may have on the aerodynamic behavior may be of the second order. Thus, by a process of abstraction of the type so often practiced in physics, it may be possible to arrive at a simplified form of the cross section for which theoretical research in the field of fluid flow may supply the fundamental data as to the magnitude and distribution of the acting air forces. Once in possession of a mathematically derived expression for the controlling air forces it is possible, as will be shown in this paper, to take into account the corrective effect of the particular geometry of the cross section. With the aerodynamic forces known, the solution of the aerodynamic problem reduces to its second phase, the investigation of the response of the structure to the action of the known resultant aerodynamic forces.

It may be in order to review the most important facts disclosed by an elaborate series of experiments on the behavior of suspension bridges under action of wind flow. These facts are:

Girder-stiffened sections develop maximum excitation when the angle of attack approaches zero (horizontal wind). The excited motion is purely vertical or purely torsional, and is identical in vibration form and frequency with one of the natural modes of the system. Vertical motions are non-catastrophic unless the ratio $\frac{d}{2\,b}$ (in which d is the girder depth, and $2\,b$ is the width of the bridge) exceeds a certain value somewhat greater than 0.2.

The wind velocity v at which critical torsional excitation starts increases considerably when the $\frac{d}{2\,b}$-ratio decreases. Experiments on section models, all vibrating with the same frequency but having various values of $\frac{d}{2\,b}$-ratios from 0.29 to 0.025, showed a continuous increase in v from 12 miles per hr to 65 miles per hr, indicating the outstanding effect of the particular shape of the suspended structure on the aerodynamic behavior of girder-stiffened bridges under wind action. The shallowest sections $\left(\frac{d}{2\,b} < 0.06\right)$ disclosed a behavior distinguished from that of the deeper sections, in that they revealed the aerodynamic characteristics of a thin flat plate.

Truss-type sections of the conventional form (closed deck) develop an optimum of stability in a horizontal wind, showing a critical velocity at zero angle of attack comparable with the critical velocity of a very shallow girder-type section or a flat plate. No self-excited vertical motion so far has been isolated in any truss-type section. However, tests indicated various cases of coupling of vertical and torsional catastrophic motion in truss-type sections of orthodox form. The period of the excited motion is distinctly different from the period of any natural mode of the vibrating system. Tests with section models having a fixed center of rotation in certain cases showed catastrophic torsional motion with a frequency always somewhat below the frequency of the natural torsional mode.

The aerodynamic properties of the truss-type section are controlled by those parts of the structure which are located near the deck level. The behavior under wind can be considerably improved by an adequate design of the chord and the adjacent part of the deck structure. Members of the stiffening frame remote from the deck appear to have little or no effect on the aerodynamic stability of the section. In fact, this type of section shows the characteristic behavior of a flat plate under horizontal wind, influenced to some extent by the particular make-up of the cross section in the vicinity of the leading or windward edge. Under upward acting wind the critical velocity decreases as the vertical angle of attack is increased.[2]

The previously enumerated experimental facts may be summarized as follows:

---

[2] "Lessons in Bridge Design Taught by Aerodynamic Studies," by F. B. Farquharson, *Civil Engineering*, August, 1946, p. 344.

(*a*) Girder-stiffened suspension bridges are aerodynamically far more vulnerable to wind action than truss-stiffened bridges. The degree of stability in a horizontal wind stream is controlled in the first instance by the over-all shape of the cross section—that is, by the $\frac{d}{2\,b}$-ratio. The excited motion is purely vertical or purely torsional and is identical in vibration form and frequency with one of the natural modes of the system.

(*b*) The stability of truss-stiffened bridges with continuous solid roadway under horizontal wind apparently is affected in but a moderate degree by the particular shape of the cross section. There are strong indications that the behavior under horizontal wind is that of a flat plate of finite thickness. Of primary importance is the observed fact that the excited motion is a coupled motion having a vertical and an angular component with a frequency distinctly different from the frequency of any of the natural modes of the system.

These facts may be considered strong evidence that there are two entirely different types of aerodynamic forces, which induce two different types of excitation under wind action.

The first type of instability may be described briefly as follows: A prismatic structure of H-shaped cross section acted on by a horizontal steady wind stream induces a periodic disturbance of the surrounding air. That disturbance originates at the windward girder (leading edge) presenting a blunt obstacle in the horizontal wind stream. The motion is a result of vortices discharged alternately from the upper and lower edges of the vertical windward faces of the structure. At a certain critical wind velocity resonance between the frequency of the disturbance and the natural frequency of the structure generates vertical or torsional oscillations with steadily increasing amplitudes—the self-excited motion. These vibrations may have restricted amplitudes or may reach catastrophic proportions. The potential dynamic instability of the structure at certain wind velocities appears here as a consequence of the particular blunt shape of the cross section.

The phenomenon observed on truss-type sections represents the second type of dynamic instability. It indicates clearly that a truss-stiffened suspension bridge with the conventional closed deck may become unstable when the structure oscillates with two degrees of freedom, vibrating vertically and in torsion simultaneously. As will be shown hereinafter, the possibility of a coupled motion constitutes the essential condition that accounts for the occurrence of catastrophic self-induced vibrations in truss-stiffened suspension bridges under action of a horizontal wind stream.

The phenomenon as characterized in the foregoing paragraph is typical of the flutter frequently observed on vibrating flat plates and airfoils. It is closely related to the specific nature of the air forces acting on a flat plate oscillating in a steady horizontal wind stream.

However, the simple concept of flutter in many cases does not suffice to explain completely the actual behavior of truss-type cross sections under wind action. The floor structure of a truss-stiffened suspension bridge, in general, cannot be considered as a completely streamlined flat plate because of the blunt

shape of the leading edge, where an alternating air force originates as a result of vortex shedding at that point. This consideration suggests the concept of an idealized cross section built up as a thin flat plate with shallow chords on both edges (Fig. 1). The vibrating structure may be thought of as acted on by air forces distributed over the plate (referred to herein as the "flat plate air forces") and an alternating resultant lift force F_v, applied to the windward chord.

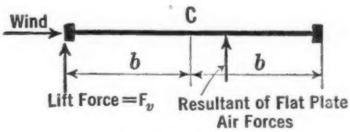


FIG. 1

The air forces acting on a horizontal flat plate freely oscillating vertically and torsionally and exposed to a horizontal wind stream are well known from theoretical investigations on the unsteady air flow around a thin flat plate. It is assumed, furthermore, that the magnitude and the nature of the lift force F_v, which depend on the particular shape of the truss chord and the adjacent part of the deck, can be determined experimentally in a rather simple manner. (It is shown hereinafter that the two unknown coefficients in the mathematical expression for F_v which determine magnitude and phase of F_v—a periodic time function—can be obtained by tests on a section model.) Theoretical solution of the problem of aerodynamic stability of the type of suspension bridges considered herein is therefore feasible, since the remaining research is confined to the investigation of the response of the structure to the given aerodynamic forces.

The following analytical study outlines a method of approach to the problem of aerodynamic instability in truss-stiffened suspension bridges and bridges with extremely shallow stiffening girders in order to obtain insight into the dynamic conditions which lead to self-induced vibrations. The analysis is based on the assumption that these two types of suspension bridge may be considered as having a suspended roadway cross section with the aerodynamic characteristics of the idealized section shown in Fig. 1. The subsequent theory supplies the rational explanation of the phenomena observed in various tests made on the full model of the preliminary design of the Tacoma Narrows Bridge with closed deck, and on a section model of the Golden Gate Bridge (San Francisco Bay, California). The preliminary design of the Tacoma Narrows Bridge had a continuous concrete slab deck. In the final design slots of open grating were introduced to improve the aerodynamic characteristics.

For the sake of simplicity, the analytical presentation of the somewhat involved theory has been divided into two parts. Sections 2 to 5 deal with the flutter theory of suspension bridges having a floor structure with flat plate characteristics. Utilizing the results of the theory of flutter outlined in these sections, the extended theory involving the flat plate having chords is developed in Section 6.

The subsequent investigation is limited to the case where the wind acts at zero angle of attack.

Notation.—The letter symbols used in this paper are defined where they first appear, in the text or by illustrations, and are assembled for convenience of reference in the Appendix. Discussers are requested to conform to this notation.

2. Free Vibrations and Self-Induced Oscillations of Systems of Two Degrees of Freedom

Free Vibrations.—A rigid bar suspended on two equal springs (Fig. 2(*a*)) and vibrating in its vertical plane represents a mechanical system of two degrees of freedom. The center of gravity is located at point C. The two principal modes of vibration having the periods p_1 and p_2 are shown in Fig. 2(*b*). The corresponding motions may be considered as rotations about points P and Q, respectively. The distances l_3 and l_4 from the center of gravity are characteristics of the system.[3] Any disturbance will cause oscillations whereby the motion of any point of the body consists of two harmonic motions of different periods. The resultant oscillations, in general, are not harmonic.

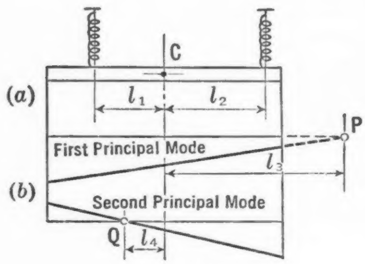


Fig. 2

For the particular case when $l_1 = l_2$, $l_3 = \infty$ and $l_4 = 0$, the two principal modes of oscillation reduce to a vertical oscillation η and to a rotation about the center of gravity, the angular oscillation ϕ. The implication from this reasoning is that the principal modes of oscillation of a suspension bridge with a symmetrical cross section, considered a system of two degrees of freedom (bending and twisting), are identical with the principal modes of vertical vibration and the principal modes of torsional oscillation of the system. (In a strict sense, since the configurations possible to an elastic structure are infinitely various, the degree of freedom of a suspension bridge actually is infinite. For the sake of simplicity it is considered that the two spectra of possible displacements (vertical and torsional) are two degrees of freedom.) Any configuration of this system may be thought of as a linear combination of a finite or infinite number of vertical and torsional modes. This implication is of primary importance in the mathematical treatment of the problem under consideration, since it may be concluded that the principal vertical and torsional modes are, in a major degree, suitable for depicting any forced or self-induced vibrations of the structure. Frequencies and displacement form of these principal modes can be conveniently determined by an energy method, outlined in its essential features in Section 3.

Self-Induced Vibrations.—In Section 3 it is demonstrated that the air forces acting on a vibrating thin flat plate are functions of the amplitudes and their derivatives with respect to time. They are, therefore, periodic forces which cause a principal change of the form of the free vibrations of the plate.

It is assumed that a periodic force F, of suitable frequency, is applied to a spring-suspended elastic flat plate vibrating under action of wind. Both ends of the long plate have hinged supports (Fig. 3). The force F is distributed along the span and is assumed of such magnitude and location that, in spite of the damping effect of the air forces at moderate wind speed, a steady-state oscilla-

[3] "Vibration Problems in Engineering," by S. Timoshenko, D. Van Nostrand Co., Inc., New York N. Y., 2d Ed., 1937, p. 131.

tion is maintained. This stable motion is characterized by a vertical and torsional component different from the corresponding modes of the free vibration, the frequencies and displacement forms depending on the wind velocity v. The two components have different frequencies and, as a result of the particular nature of the air forces, show a difference of phase; the motion is not harmonic.

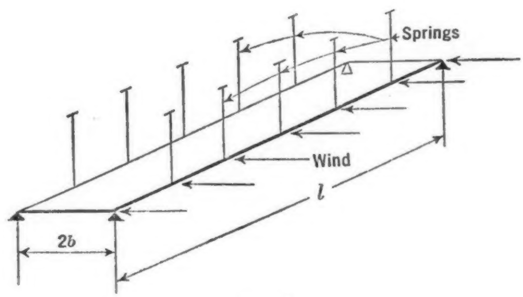


Fig. 3

With steadily increasing wind velocity the external force F, necessary to maintain the motion, at first increases and then decreases until a point is reached where the air forces alone sustain a constant amplitude of the oscillation. The corresponding velocity v is called the critical velocity or the flutter velocity. In Fig. 4, F is plotted against v. At critical wind speed, v_c, the two components of the coupled motion have the same frequency, but show a difference of their phase angles. This difference of the phase angles accounts for the fact that above the critical velocity the structure absorbs energy from the air, with the effect that the amplitudes of small initial oscillations increase steadily until they reach dangerous proportions.

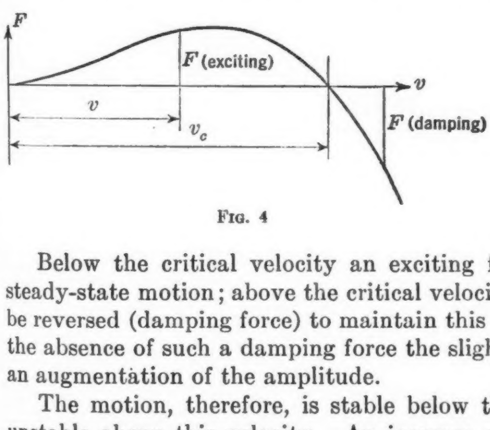


Fig. 4

Below the critical velocity an exciting force is necessary to maintain a steady-state motion; above the critical velocity the direction of the force must be reversed (damping force) to maintain this steady-state motion (Fig. 4). In the absence of such a damping force the slightest increase of v above v_c causes an augmentation of the amplitude.

The motion, therefore, is stable below the critical velocity and becomes unstable above this velocity. An increase of v beyond that limit leads to a catastrophic response of the vibrating structure. The specification of the condition of instability (that is, the determination of the critical velocity and the flutter frequency) is the problem with which this paper deals.

3. Air Forces Acting on a Vibrating Flat Plate

The determination of the dynamic air forces acting on a flat plate or on an airfoil which oscillates vertically and torsionally in a steady air stream has been treated by several authors in the last two decades. Expressions for the air forces suitable for the following investigation were derived by T. Theodorsen.[4]

[4] "General Theory of Aerodynamic Instability and the Mechanism of Flutter," by T. Theodorsen, National Advisory Committee for Aeronautics, *Technical Report No. 496*, Washington, D. C., 1935.

Equivalent equations have been developed independently by R. Kassner and H. Fingado[5] and by Theodor von Kármán, M. ASCE, and W. R. Sears.[6] The Theodorsen solution is based on the assumption that the vibrating flat plate is infinitely thin and that the air flow is two dimensional. Under the latter assumption the equations apply only to cases where the object under wind action is sufficiently extended perpendicularly to the air flow so that the altered conditions at the ends influence the total effect of the air forces only to a negligible degree. This condition is satisfied to a large extent in the case of suspension bridges.

The Theodorsen equations were derived for small harmonic oscillations about the position of equilibrium, composed of simultaneous vertical and torsional vibrations of constant amplitudes. Consequently, they apply solely to the narrow zone of transition from stable to unstable motion, which may be considered a sustained harmonic oscillation of constant amplitude. Therefore, the Theodorsen expressions for the dynamic air forces suffice to solve the problem of self-excitation from its practical viewpoint—that is, the specification of the conditions of instability or, more exactly, the determination of the critical wind velocity and the characteristics of the motion at that wind speed.

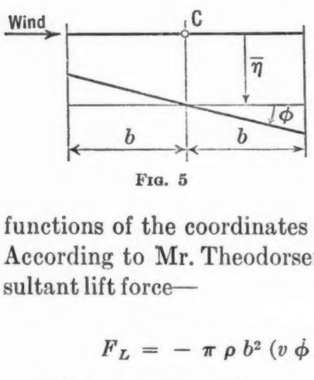


FIG. 5

Consider a long flat plate having the width $2\,b$, and vibrating vertically and torsionally about its position of equilibrium; $\bar{\eta}$ is the vertical or bending amplitude of the center point C, downward displacements being positive; ϕ is the angle of rotation, the torsional amplitude, counted positive when the plate rotates clockwise (Fig. 5). The air forces are expressed as functions of the coordinates $\bar{\eta}$ and ϕ and their derivatives with respect to time. According to Mr. Theodorsen, a unit length of the plate is acted on by the resultant lift force—

$$F_L = -\pi\,\rho\,b^2\,(v\,\dot{\phi} + \ddot{\bar{\eta}}) - 2\,\pi\,\rho\,b\,v\,C(k)\left(v\,\phi + \dot{\bar{\eta}} + \frac{b}{2}\dot{\phi}\right)\ldots\ldots(1)$$

—and the resultant lift moment—

$$M_L = -\pi\,\rho\,b^2\left(v\,\frac{b}{2}\dot{\phi} + \frac{b^2}{8}\ddot{\phi}\right) + \pi\,\rho\,b^2\,v\,C(k)\left(v\,\phi + \dot{\bar{\eta}} + \frac{b}{2}\dot{\phi}\right)\ldots(2)$$

—in which ρ is the mass density of air (assumed 0.00238 slugs); $\dot{\bar{\eta}}$ and $\dot{\phi}$ are the first derivatives with respect to time of the coordinates $\bar{\eta}$ and ϕ, respectively (velocities); $\ddot{\bar{\eta}}$ and $\ddot{\phi}$ are the second derivitives with respect to time of the coordinates $\bar{\eta}$ and ϕ, respectively (accelerations); and $C(k)$ is a complex function of the dimensionless ratio $k = \dfrac{\omega\,b}{v}$, ω being the flutter frequency. Positive F_L denotes a downward acting force; positive M_L, a clockwise acting moment.

[5] "The Two-Dimensional Problem of Wind Vibration," by R. Kassner and H. Fingado, *Journal*, Royal Aeronautical Soc., Vol. 41, 1937, p. 921.

[6] "Airfoil Theory for Non-Uniform Motion," by Theodor von Kármán and W. R. Sears, *Journal of the Aeronautical Sciences*, August, 1938, p. 379.

Mr. Theodorsen developed his solution for a system of three degrees of freedom considering the air flow over an airfoil with an aileron. Eqs. 1 and 2 are derived from the Theodorsen equations[7] by omitting all terms pertaining to the aileron and setting $a = 0$ and $h = \bar{\eta}$.

The terms in Eqs. 1 and 2 containing $\ddot{\bar{\eta}}$ and $\ddot{\phi}$ represent the inertia effect of a cylinder of air having a diameter equal to the width $2\,b$. These terms could be easily combined with the terms expressing the inertia force of the vibrating body in the equations of motion. It is apparent, however, that, in the case of a vibrating suspension bridge, the inertia effect $\pi\,\rho\,b^2\,\ddot{\bar{\eta}}$ in Eq. 1 is very small in comparison with the inertia force $m\,\ddot{\bar{\eta}}$ of the vibrating structure, m being the mass of the bridge per unit length. The same holds true for the inertia term in Eq. 2. Accordingly, these terms may be disregarded without substantial error. Rearranging, Eqs. 1 and 2 may be written

$$F_L = -\,2\,\pi\,\rho\,b\,v^2 \left\{ C(k)\left(\phi + \frac{\dot{\eta}\,b}{v}\right) + [1 + C(k)]\,\frac{b}{2}\,\frac{\dot{\phi}}{v} \right\} \ldots\ldots (3)$$

$$M_L = \pi\,\rho\,b^2\,v^2 \left\{ C(k)\left(\phi + \frac{\dot{\eta}\,b}{v}\right) + [1 - C(k)]\,\frac{b}{2}\,\frac{\dot{\phi}}{v} \right\} \ldots\ldots\ldots (4)$$

in which the dimensionless quantity $\eta = \frac{\bar{\eta}}{b}$ has been introduced. Both ϕ and η are now dimensionless coordinates.

The function $C(k)$ is a complex function having the form:

$$C(k) = F(k) - i\,G(k) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

in which $F(k)$ and $G(k)$ represent transcendental functions of k and $i = \sqrt{-1}$;

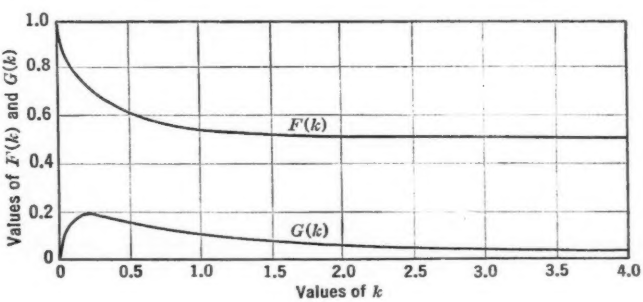


Fig. 6

$F(k)$ and $G(k)$ are plotted against k in Fig. 6. It may be noted that $F(k)$ approaches the value 0.5 and $G(k)$ approaches 0 when k approaches ∞. The critical value of k corresponding with the critical velocity v_c, as far as suspension bridges are concerned, lies below $k = 1$. For that reason, Table 1, which

[7] "General Theory of Aerodynamic Instability and the Mechansim of Flutter," by T. Theodorsen, National Advisory Committee for Aeronautics, *Technical Report No. 496*, Washington, D. C., 1935, p. 419, equations XVIII and XX.

shows the numerical values of $F(k)$ and $G(k)$ at appropriate intervals, is restricted to values of k between 0 and 1.[8]

It may be proper to discuss briefly the complex form of the function $C(k)$ which simply signifies that $C(k)$ may be thought of as composed of two components, different as to their magnitude, one component a quarter of the period

TABLE 1.—VALUES OF FUNCTIONS $F(k)$ AND $G(k)$ FROM $k = 0$ TO $k = 1$

k (1)	$F(k)$ (2)	$G(k)$ (3)	k (1)	$F(k)$ (2)	$G(k)$ (3)	k (1)	$F(k)$ (2)	$G(k)$ (3)	k (1)	$F(k)$ (2)	$G(k)$ (3)
0	1.0000	0.0000	0.18	0.7443	0.1887	0.36	0.6390	0.1709	0.60	0.5788	0.1378
0.02	0.9637	0.0752	0.20	0.7276	0.1886	0.38	0.6317	0.1679	0.65	0.5713	0.1319
0.04	0.9267	0.1160	0.22	0.7125	0.1877	0.40	0.6250	0.1650	0.70	0.5648	0.1264
0.06	0.8920	0.1426	0.24	0.6989	0.1862	0.42	0.6187	0.1621	0.75	0.5591	0.1213
0.08	0.8604	0.1604	0.26	0.6865	0.1842	0.44	0.6130	0.1592	0.80	0.5541	0.1165
0.10	0.8319	0.1723	0.28	0.6753	0.1819	0.46	0.6076	0.1563	0.85	0.5498	0.1121
0.12	0.8063	0.1801	0.30	0.6650	0.1793	0.48	0.6026	0.1535	0.90	0.5459	0.1078
0.14	0.7834	0.1849	0.32	0.6556	0.1766	0.50	0.5979	0.1507	0.95	0.5425	0.1039
0.16	0.7628	0.1876	0.34	0.6469	0.1738	0.55	0.5866	0.1444	1.00	0.5394	0.1003

ahead in time with respect to the other. The functions $C(k)$ affect the time factor in η and ϕ in a different degree, thus producing a difference in the phase angles of the vertical and torsional oscillations, as pointed out in Section 2.

Substituting, finally,

$$C(k) = f_1 \qquad (6a)$$

$$1 + C(k) = f_2 \qquad (6b)$$

$$1 - C(k) = f_3 \qquad (6c)$$

and

$$2\,\pi\,\rho\,b = s \qquad (6d)$$

Eqs. 3 and 4 are preferably written in the form:

$$F_L = -\,s\,v^2 \left[f_1 \left(\phi + \frac{b}{v}\,\dot{\eta} \right) + f_2\,\frac{b}{2\,v}\,\dot{\phi} \right] \qquad (7)$$

$$M_L = \frac{s\,b\,v^2}{2} \left[f_1 \left(\phi + \frac{b}{v}\,\dot{\eta} \right) - f_3\,\frac{b}{2\,v}\,\dot{\phi} \right] \qquad (8)$$

4. THE DIFFERENTIAL EQUATIONS OF SELF-EXCITED VIBRATIONS IN SUSPENSION BRIDGES DUE TO THE FLAT PLATE AIR FORCES

Establishment of the differential equations of motion is the aim of this section, the final solution of the flutter problem being treated in Section 5.

Some introductory explanations may facilitate understanding of the peculiarity of the problem of self-excitation under wind action.

A self-induced vibration may be defined as a phenomenon in which the alternating forces furnishing the energy to the oscillations are controlled by the motion itself. When motion ceases, the alternating forces disappear. This type of periodic motion is distinctly different from a forced vibration where the

[8] "The Two-Dimensional Problem of Wind Vibrations," by R. Kassner and H. Fingado, *Journal*, Royal Aeronautical Soc., Vol. 41, 1937, p. 945.

periodic forces exist independently of the motion and persist when the motion is stopped.

A self-induced oscillation is described by a system of homogeneous linear differential equations, because the acting air forces are functions of the amplitudes and their derivatives with respect to time. These equations may be interpreted as the differential equations of a free, but damped, oscillation of a mechanical system. Because of the peculiarity of the dynamic air forces, resulting from the nonstationary flow around the oscillating structure, the damping effect of these forces may be positive, so that any initial oscillation decreases and finally disappears; or the forces may be negative, in which case the initial amplitude increases steadily. In the latter case they are referred to as self-induced vibrations. The point at which damping changes from positive to negative determines the incipient state of dynamic instability. Above this critical point—associated with a certain wind velocity—the motion shows, in general, disproportionately large increases of the amplitudes at slight increases of the wind velocity.

The solution of this problem—namely, the determination of the border line between stable and unstable motion—is facilitated by the following facts: The motion at critical wind velocity is a steady-state motion. The vertical and torsional vibrations composing the coupled motion have the same frequency ω as was outlined in Section 2, so that the motion is harmonic. Applying the Theodorsen expressions for the air forces, the vibration problem can be solved, at least to the point that the condition of instability can be specified. It is worthy of note that the homogeneous linear differential equations which define the motion at the incipient state of instability are not suitable for determining the character of the motion when the critical velocity is exceeded.

Principal Modes of Free Vibrations of Suspension Bridges.—Because investigation of the condition of flutter basically is tied to the natural modes of vibration, it appears necessary to outline briefly the essential features of an energy method that permits specification of frequencies and displacement forms of the natural modes of suspension bridges without the use of a complex mathematical framework and without cumbersome numerical calculations. The method affords an easy and a quick solution of the problem, even in cases where the effect of additional structural devices, such as cable ties, diagonal stays, double system of lateral bracing, and the like, is involved.

The method discussed hereinafter is an approximation method and is based on a minimum principle of dynamics. The application of such a minimum condition practically reduces the determination of the frequencies to the solution of an algebraic equation of second or third degree.

A freely oscillating and undamped suspension bridge represents a conservative mechanical system. No energy is imparted to the structure from any external source and no energy is dissipated by any kind of damping. The frequency ω and the configuration of any natural mode of vibration of such a conservative system are determined by the minimum condition:

$$T - V = \text{minimum} \qquad (9)$$

in which T denotes the maximum value of the kinetic energy of the entire structure, a condition occurring when the structure passes its equilibrium position; and V denotes the maximum potential energy (elastic and gravitational) accumulated in the structure when it occupies its extreme position—T and V are homogeneous quadratic functions of the amplitude η and its derivatives with respect to x. (Since η varies along the span it is a function of the abscissa x.) Assuming small oscillations about the position of equilibrium for the two types of motion, T and V can be expressed as follows:

Vertical Vibrations.—

$$T = \frac{m\,\omega^2}{2}\int_L \eta^2\,dx \qquad (10)$$

and

$$V = \frac{E_f\,I_f}{2}\int_L (\eta'')^2\,dx + \frac{H_w}{2}\int_L (\eta')^2\,dx + \frac{4\,h}{L^2_m}\,H_i\int_L \eta\,dx \qquad (11)$$

in which L_m represents the length of the main span; h is the cable sag of the main span; m is the mass of the bridge per unit length; E_f is the modulus of elasticity of the stiffening frames; I_f is the total moment of inertia of the stiffening frames; H_w is the cable tension (all cables) due to dead weight w; and H_i is the cable tension (all cables) due to the inertia forces. The integrals extend over the total length of the bridge L.

The cable tension H_i is connected with η by the linearized cable equation:

$$\frac{L_c}{E_c\,A_c}\,H_i - \frac{8\,h}{L^2_m}\int_L \eta\,dx = 0 \qquad (12)$$

in which A_c and E_c denote the cross-sectional area and the modulus of elasticity of the cables; and the length L_c is defined by

$$L_c = \int\left(\frac{ds}{dx}\right)^2 ds \qquad (13)$$

—the integral extending from anchorage to anchorage of the cables.

Eliminating H_i by use of Eq. 12 gives the following expression for the energy difference,

$$T - V = \frac{m\,\omega^2}{2}\int_L \eta^2\,dx - \frac{E_f\,I_f}{2}\int_L (\eta'')^2\,dx - \frac{H_w}{2}\int_L (\eta')^2\,dx$$

$$- \frac{32\,h^2}{L^4_m}\,\frac{E_c\,A_c}{L_c}\left(\int_L \eta\,dx\right)^2 \qquad (14)$$

Torsional Vibrations.—If ϕ is the angle of rotation and r is the mass radius of gyration of the cross section of the bridge, immediately $T - V$ may be derived from Eq. 14 by substituting

$$\eta^2 = r^2\,\phi \qquad (15a)$$

in the first term and substituting

$$\eta = b\,\phi \qquad (15b)$$

in the other terms. Thus,

$$T - V = \frac{m\,r^2\,\omega^2}{2}\int_L \phi^2\,dx - \frac{E_f\,I_f\,b^2}{2}\int_L (\phi'')^2\,dx - \frac{H_w\,b^2}{2}\int_L (\phi')^2\,dx$$
$$- \frac{32\,h^2\,b^2}{L^4{}_m}\frac{E_c\,A_c}{L_c}\left(\int_L \phi\,dx\right)^2 \qquad (16)$$

Eqs. 14 and 16 apply to suspension bridges of the conventional type. When structural devices are installed to augment vertical or torsional rigidity, Eqs. 14 and 16 must be extended by terms indicating the amount of energy—expressed as a function of η or ϕ—accumulated in the added structural parts.

The condition expressed by Eq. 9 requires that under all possible configurations of the vibrating structure that configuration η or ϕ must be found which makes Eq. 14 or Eq. 16 have a minimum value. A general method for solving such minimum problems by approximation has been devised by W. Ritz.[9] This method is based on the following reasoning:

If η could be represented by a linear combination of a sequence of suitable functions $\Phi_1, \Phi_2, \cdots, \Phi_n$ satisfying certain conditions which will be specified subsequently, and $a_1, a_2, \cdots, a_n$ represent a corresponding set of parameters, then η may be assumed as having the form:

$$\eta = a_1\,\Phi_1 + a_2\,\Phi_2 + \cdots + a_n\,\Phi_n \qquad (17)$$

The nature of the problem dealt with imposes an important restriction on the form of the functions Φ. The displacements defined by Φ must be possible displacements of the elastic system under consideration and, as a logical consequence, must satisfy the conditions of restraint—that is, the end conditions. Thereafter the form of the functions Φ, with certain limitations, remains arbitrary. These limitations are governed only by the requirement that the values of η are to be determined with sufficient accuracy by as small a number of parameters as possible.

Introducing Eq. 17 into the energy equation (Eq. 14) and performing the integrations indicated leads to an expression giving $T - V$ as a quadratic junction of the n parameters a. Inasmuch as η must satisfy Eq. 9, the parameters $a_1, a_2, \ldots, a_n$ are determined by the condition that $T - V$ becomes a minimum. Thus,

$$\frac{\partial(T - V)}{\partial a_1} = 0,\ \frac{\partial(T - V)}{\partial a_2} = 0,\ \cdots,\ \frac{\partial(T - V)}{\partial a_n} = 0 \qquad (18)$$

The first derivative of a quadratic function is a linear function. Eqs. 18 therefore represent a system of n homogeneous linear equations from which the parameters a can be determined. This system of equations does not have any solution unless the determinant Δ of the coefficients is equal to zero. Thus,

$$\Delta = 0 \qquad (19)$$

which is an equation of the nth degree of the unknown ω^2, represents the fre-

[9] "Über eine neue Methode zur Lösung gewisser Varitionsprobleme der mathematischen Physik," by W. Ritz, *Journal für die reine und angewandte Mathematik*, Vol. 135, 1909, p. 1.

quency equation that determines ω. Eq. 19 yields n different roots ω^2 associated with the first, second, $\cdots$, nth mode of vibration. In general, the degree of accuracy decreases somewhat for the higher modes. By substituting the numerical value of any of the roots of Eq. 19 into the linear equations (Eqs. 18) the relative values of the parameters a, expressed by one arbitrary parameter, can be calculated and the configuration of the modes of vibration associated with that root thereby specified.

The Ritz method, when based upon a proper set of functions Φ, furnishes a sequence of parameters a, rapidly converging to the limit zero, so that a few terms of the series (Eq. 17) suffice to determine ω and η with the necessary degree of accuracy. Success or failure in applying the Ritz method depends largely on the proper choice of the functions Φ. Such an appropriate set of functions can be found in the sequence of displacement forms representing the solution of a related vibration problem having the same boundary conditions. For instance, the functions $\sin \frac{\pi x}{L_m}$, $\sin \frac{2 \pi x}{L_m}$, $\cdots$, $\sin \frac{n \pi x}{L_m}$ represent the principal modes of vibration of a hinged beam (the stiffening frame without cable) It may be expected, therefore, that these functions will prove to be that sequence of suitable Φ-functions on which the approximate solution of the problem of free vibration of suspension bridges with hinged stiffening frame can be based.

Equation of Motion of Self-Induced Vibrations.—The complexity of the vibrating system and the involved form of the mathematical expressions for the air forces render it difficult to devise a direct solution of the dynamic problem of self-excitation by starting from the differential equation of small vibrations. However, another method is available to deal successfully with the somewhat involved problem.

The solution of the problem of the free vibrations of suspension bridges as discussed hereinbefore has been greatly simplified by using the Ritz method, in which the parameters a represent a particular type of the so-called "generalized coordinates," a concept which has proved to be an important tool in connection with the Lagrangian equations of motion for dynamic problems.

Following the pattern established by the energy method the assumption can be made that the configuration of the elastic system at flutter speed may be depicted by an appropriately chosen set of functions Φ. It may be assumed, therefore, that the two vibration coordinates, η and ϕ, the space-time functions which define the configuration of the vibrating system at any instant, are satisfactorily expressed by two finite series of the form:

$$\eta = q_{\eta 1} \Phi_{\eta 1} + q_{\eta 2} \Phi_{\eta 2} + \cdots + q_{\eta n} \Phi_{\eta n} \qquad (20a)$$

and

$$\phi = q_{\phi 1} \Phi_{\phi 1} + q_{\phi 2} \Phi_{\phi 2} + \cdots + q_{\phi n} \Phi_{\phi n} \qquad (20b)$$

in which the coefficients q are products $(u \, \psi)$, u being a parameter having the dimension of η or ϕ, respectively, and ψ representing a time function. The quantities q are the generalized coordinates. In combination with the chosen space functions Φ they determine the configuration of the vibrating system at any moment.

An elastic structure representing a continuous mechanical system has an infinite number of degrees of freedom; that is, an infinite number of coordinates is necessary to describe the configuration of the system. Introduction of generalized coordinates has the effect of expressing the configuration of such a system by a small number of appropriate coordinates. Thus, the degree of freedom of the system, at least as far as the mathematical treatment is concerned, is reduced to the number of generalized coordinates.

There is no difficulty concerning the choice of the Φ-functions that are to be introduced in the subsequent analysis of the binary motion of suspension bridges. The principal modes of the vertical and the torsional vibrations of that suspension bridge whose stability conditions are to be specified offer themselves as a suitable set of Φ-functions, as was indicated in Section 2 in discussing the free vibration of a suspension bridge considered as a system of two degrees of freedom. Since the first occurrence of dynamic instability logically is associated with the lowest frequencies, corresponding to a symmetric or an asymmetric vibration, the set of Φ-functions always must include the first symmetric or the first asymmetric principal modes of vertical and torsional vibrations.

Lagrange's Equations of Motion.—In the previous discussion of the free vibrations a minimum principle of mechanics was utilized in order to obtain the equations from which the values of the parameters a (generalized coordinates) determining the various modes of free vibration could be derived. In dealing with the problem of self-excitation resort must be had to a more general form of the equation of motion—the Lagrangian equations for small motion. The condition of Eq. 9, on which the theory of free vibration could be based, applies only to a conservative mechanical system represented, for instance, by an undamped, freely vibrating structure. However, despite the fact that the motion under wind action at flutter speed may be considered a sustained motion, the oscillating system, in reality, is a nonconservative one. Energy input from the surrounding air takes place during one part of the cycle, whereas an equal amount of energy is dissipated during another part of the cycle.

The subsequent theory of self-excitation in suspension bridges utilizes the fundamentals of a concept which already has been applied to the flutter problem in aircraft design.[10]

The Lagrangian equations for small motion of a mechanical system about a position of equilibrium are:[11]

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_j} + \frac{\partial V}{\partial q_j} - Q_j = 0 \qquad (21)$$

in which $j = 1, 2, \cdots, n$; q_j are the n generalized coordinates; $\dot{q}_j$ is the first derivative of q_j with respect to time (velocity); T is the kinetic energy of the oscillating system and is a quadratic function of the generalized coordinates; V is the total potential energy of the system and is a quadratic function of the generalized coordinates; and Q_j is the generalized force corresponding to the generalized coordinate q_j. The generalized coordinates q_j have, as previously

[10] "General Approach to the Flutter Problem," by S. J. Loring, *Journal*, S.A.E., August, 1941, p. 345.

[11] "Mathematical Methods in Engineering," by Theodor von Kármán and Maurice A. Biot, McGraw-Hill Book Co., Inc., New York and London, 1940.

stated, the form of products—

$$q_j = u_j\,\psi_j \qquad (22)$$

The parameters u_j may be considered in the following dimensionless quantities since η and ϕ have been defined as magnitudes of zero dimension (see Section 3). The symbol ψ_j is the time function $e^{\omega_j t}$, in which e is the base of the Naperian logarithms; ω_j is a complex number; and t represents time. For the system under consideration the vertical amplitudes η and the angle of rotation ϕ are expressed by the two series (Eqs. 20) involving, together, 2 n generalized coordinates q_j. There are 2 n equations of the type of Eq. 21. Accordingly, a set of 2 n values of q_j is determined by 2 n equations.

Application of Lagrange's Equations to the Flutter Problem of Suspension Bridges.—The derivation of the equations of motion from the basic Eq. 21 will be shown conveniently in applying the Lagrangian equations to the most simplified case, in which it is assumed that a set of only two generalized coordinates leads to a sufficiently accurate solution. It is easily recognized that the coupled motion associated with a flutter frequency, which always lies between the two frequencies of the first vertical and the first torsional principal modes of free vibration (either symmetric or asymmetric), is controlled by these two frequencies. Therefore, the displacement form of the two components of the coupled motion cannot be too far different from the configuration of the first vertical and first torsional principal mode, respectively. The next higher modes may have only a corrective effect on the computed critical velocity and flutter frequency.

The following analysis reveals that, in the case of the conventional type of suspension bridges, in which the torsional rigidity of the bridge structure is determined only by the vertical rigidity of the compound system composed of cables and trusses, a set of two generalized coordinates suffices to arrive at an exact solution of the problem. The particular reason for that behavior, as will be shown subsequently, is that the displacement forms Φ_η and Φ_ϕ associated with the chosen generalized coordinates are identical ($\Phi_\eta = \Phi_\phi$) for each principal mode. This simple relationship does not hold true for bridges whose torsional rigidity is materially affected by the torsional resistance of the towers or by installation of a double lateral system or other devices. However, even in these cases, restriction to two coordinates leads to an approximate solution of the flutter problem which is sufficiently accurate.

If a three-span suspension bridge oscillating in a symmetric configuration under action of wind is considered and it is assumed, in accordance with the foregoing explanation, that

$$\eta = q_1\,\Phi_1 \qquad (23a)$$

and

$$\phi = q_2\,\Phi_2 \qquad (23b)$$

in which Φ_1 represents the first symmetric principal mode of vertical oscillation and Φ_2 represents the corresponding mode of angular vibration, the kinetic energy of the oscillating system is given by

$$T = \frac{m\,b^2}{2}\int_L \dot{\eta}^2\,dx + \frac{m\,r^2}{2}\int_L \dot{\phi}^2\,dx \qquad (24)$$

In Eq. 24, the integrals extend over the three spans with a total length L. Substituting Eqs. 23 in Eq. 24 gives

$$T = \frac{m\,b^2}{2}\,\dot{q}^2{}_1 \int_L \Phi^2{}_1\,dx + \frac{m\,r^2}{2}\,\dot{q}^2{}_2 \int_L \Phi^2{}_2\,dx \qquad (25)$$

Differentiation of Eq. 25 with respect to $\dot{q}_1$ and $\dot{q}_2$, respectively, and again, differentiation with respect to t, gives

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_1} = m\,b^2\,\ddot{q}_1 \int_L \Phi^2{}_1\,dx \qquad (26a)$$

and

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_2} = m\,r^2\,\ddot{q}_2 \int_L \Phi^2{}_2\,dx \qquad (26b)$$

These are the first terms in each of the two Lagrangian equations corresponding with the generalized coordinates q_1 and q_2; $\ddot{q}$ denotes the second derivative with respect to time, or the acceleration.

The second term in each of the two Lagrangian equations is dependent on the potential energy. Thus,

$$V = \frac{E\,I\,b^2}{2}\left[\int_L (\eta'')^2\,dx + \int_L (\phi'')^2\,dx\right] - \frac{H_w\,b^2}{2}\left[\int_L \eta\,\eta''\,dx + \int_L \phi\,\phi''\,dx\right] + \frac{K}{2}\left[\left(\int_L \eta\,dx\right)^2 + \left(\int_L \phi\,dx\right)^2\right] \qquad (27)$$

in which

$$K = \frac{64\,h^2\,b^2}{L^4{}_m}\,\frac{E_c\,A_c}{L_c} \qquad (28)$$

Substitution of Eqs. 23 in Eq. 27 gives

$$V = \frac{E\,I\,b^2}{2}\left[q^2{}_1 \int (\Phi''{}_1)^2\,dx + q^2{}_2 \int_L (\Phi''{}_2)^2\,dx\right] - \frac{H_w\,b^2}{2}\left[q^2{}_1 \int_L \Phi_1\,\Phi''{}_1\,dx + q^2{}_2 \int_L \Phi_2\,\Phi''{}_2\,dx\right] + \frac{K}{2}\left[q^2{}_1\left(\int_L \Phi_1\,dx\right)^2 + q^2{}_2\left(\int_L \Phi_2\,dx\right)^2\right] \qquad (29)$$

from which

$$\frac{\partial V}{\partial q_1} = q_1\left[E\,I\,b^2 \int (\Phi''{}_1)^2\,dx - H_w\,b^2 \int_L \Phi_1\,\Phi''{}_1\,dx + K\left(\int_L \Phi_1\,dx\right)^2\right] \qquad (30a)$$

and

$$\frac{\partial V}{\partial q_2} = q_2\left[E\,I\,b^2 \int_L (\Phi''{}_2)^2\,dx - H_w\,b^2 \int_L \Phi_2\,\Phi''{}_2\,dx + K\left(\int_L \Phi_2\,dx\right)^2\right] \qquad (30b)$$

Eq. 21, therefore, takes the form:

$$m\, b^2\, \ddot{q}_1 \int_L \Phi^2{}_1\, dx + q_1 \left[E\, I\, b^2 \int_L (\Phi''{}_1)^2\, dx - H_w\, b^2 \int_L \Phi_1\, \Phi''{}_1\, dx + K \left(\int_L \Phi_1\, dx \right)^2 \right] = Q_1 \quad \text{(31a)}$$

and

$$m\, r^2\, \ddot{q}_2 \int_L \Phi^2{}_2\, dx + q_2 \left[E\, I\, b^2 \int_L (\Phi''{}_2)^2\, dx - H_w\, b^2 \int_L \Phi_2\, \Phi''{}_2\, dx + K \left(\int_L \Phi_2\, dx \right)^2 \right] = Q_2 \quad \text{(31b)}$$

Before determining the generalized forces Q_1 and Q_2, Eqs. 31 may be simfied by introducing the frequencies ω_1 and ω_2 associated with the two principal modes Φ_1 and Φ_2. Assuming $Q_1 = 0$ and $Q_2 = 0$ in Eqs. 31, two homogeneous equations, independent of each other, are derived which define the vertical and the torsional free vibrations of the system. Consequently, the solutions

$$q_1 = e^{i\omega_1 t} \quad \text{(32a)}$$

and

$$q_2 = e^{i\omega_2 t} \quad \text{(32b)}$$

must satisfy these equations.

By introducing Eqs. 32 and substituting,

$$\ddot{q}_1 = -\,\omega^2{}_1\, e^{i\omega_1 t} \quad \text{(33a)}$$

and

$$\ddot{q}_2 = -\,\omega^2{}_2\, e^{i\omega_2 t} \quad \text{(33b)}$$

in the two homogeneous equations, gives

$$-\,m\, b^2\, \omega^2{}_1 \int_L \Phi^2{}_1\, dx + E\, I\, b^2 \int_L (\Phi''{}_1)^2\, dx - H_w\, b^2 \int_L \Phi_1\, \Phi''{}_1\, dx + K \left(\int_L \Phi_1\, dx \right)^2 = 0 \quad \text{(34a)}$$

and

$$-\,m\, r^2\, \omega^2{}_2 \int_L \Phi^2{}_2\, dx + E\, I\, b^2 \int_L (\Phi''{}_2)^2\, dx - H_w\, b^2 \int_L \Phi_2\, \Phi''{}_2\, dx + K \left(\int_L \Phi_2\, dx \right)^2 = 0 \quad \text{(34b)}$$

When using these relations Eqs. 31 may be written in the more condensed form:

$$(\ddot{q}_1 + \omega^2{}_1\, q_1) \int_L \Phi^2{}_1\, dx = \frac{Q_1}{m\, b^2} \quad \text{(35a)}$$

and

$$(\ddot{q}_2 + \omega^2{}_2\, q_2) \int_L \Phi^2{}_2\, dx = \frac{Q_2}{m\, r^2} \ldots\ldots\ldots\ldots\ldots (35b)$$

So far, it has been tacitly assumed that the rigidity of the structure is due only to the combined effect of cables and stiffening frames. Suppose now that Eqs. 31 have been extended by additional terms related to that part of the elastic energy which is accumulated in reinforcing structural elements, such as tower stays, center stays, and the like. The same additional terms also appear in Eqs. 34, so that the end results are equations of motion of the form of Eqs. 35. Eqs. 35, therefore, apply to any type of suspension bridge.

In determining the generalized forces Q_1 and Q_2, the external forces F_L and M_L are defined by Eqs. 7 and 8 as functions of the coordinates η and ϕ and their derivatives with respect to t. The work done by these forces during a virtua change of the configuration of the system defined by the virtual displacements $b\, \delta\eta$ and $\delta\phi$, may be expressed as

$$\delta W = \int_L (F_L\, b\, \delta\eta + M_L\, \delta\phi)\, dx \ldots\ldots\ldots\ldots\ldots (36)$$

On the other hand, δW can also be determined as the work done by the generalized forces Q_1 and Q_2 due to the virtual displacements δq_1 and δq_2. Thus,

$$\delta W = Q_1\, \delta q_1 + Q_2\, \delta q_2 \ldots\ldots\ldots\ldots\ldots (37)$$

The variations δq_1 and δq_2 are thought of as due to a variation of the dimensionless parameters u_1 and u_2 connected with the coordinates q_1 and q_2.

Expressing F_L, M_L, $\delta\eta$, and $\delta\phi$ in Eq. 36 by the generalized coordinates q_1 and q_2,

$$\delta W = \delta q_1 \int_L R_1\, dx + \delta q_2 \int_L R_2\, dx \ldots\ldots\ldots\ldots\ldots (38)$$

in which R_1 and R_2 are expressions depending on the air forces F_L and M_L. Equating the coefficients of δq_1 and δq_2 in Eqs. 37 and 38 furnishes the expressions for the generalized forces. Accordingly,

$$Q_1 = \int_L R_1\, dx \ldots\ldots\ldots\ldots\ldots (39a)$$

and

$$Q_2 = \int_L R_2\, dx \ldots\ldots\ldots\ldots\ldots (39b)$$

Using Eqs. 7 and 8 and substituting

$$\eta = q_1\, \Phi_1 \ldots\ldots\ldots\ldots\ldots (40a)$$

and

$$\phi = q_2\, \Phi_2 \ldots\ldots\ldots\ldots\ldots (40b)$$

The terms in Eq. 36 become

$$\int_L F_L\, b\, \delta\eta\, dx = -\, s\, b\, v^2 \int_L \left[f_1 \left(q_2\, \Phi_2 + \frac{b}{v}\, \dot{q}_1\, \Phi_1 \right) + f_2\, \frac{b}{2\, v}\, \dot{q}_2\, \Phi_2 \right] \Phi_1\, \delta q_1\, dx \ldots\ldots (41a)$$

and

$$\int_L M_L\, \delta\phi\, dx = \frac{s\, b\, v^2}{2} \int_L \left[f_1 \left(q_2\, \Phi_2 + \frac{b}{v}\, \dot{q}_1\, \Phi_1 \right) - f_3\, \frac{b}{2\, v}\, \dot{q}_2\, \Phi_2 \right] \Phi_2\, \delta q_2\, dx \ldots\ldots (41b)$$

and, finally,

$$\delta W = -\, s\, b\, v^2\, \delta q_1 \int_L \left[f_1 \left(q_2\, \Phi_1\, \Phi_2 + \frac{b}{v}\, \dot{q}_1\, \Phi^2{}_1 \right) + f_2\, \frac{b}{2\, v}\, \dot{q}_2\, \Phi_1\, \Phi_2 \right] dx$$

$$+ \frac{s\, b\, v^2}{2}\, \delta q_2 \int_L \left[f_1 \left(q_2\, \Phi^2{}_2 + \frac{b}{v}\, \dot{q}_1\, \Phi_1\, \Phi_2 \right) - f_3\, \frac{b}{2\, v}\, \dot{q}_2\, \Phi^2{}_2 \right] dx \ldots\ldots (42)$$

In Eq. 42 the terms within the brackets when multiplied by $-\, s\, b\, v^2$ and $\frac{s\, b\, v^2}{2}$, respectively, represent the magnitudes R_1 and R_2 of Eq. 38 and define the generalized forces Q_1 and Q_2 according to Eqs. 39.

The differential equations of motion (Eqs. 35) therefore assume the form:

$$(\ddot{q}_1 + \omega^2{}_1\, q_1) \int_L \Phi^2{}_1\, dx + \frac{s\, b\, v^2}{m\, b^2} \left(f_1\, q_2 \int_L \Phi_1\, \Phi_2\, dx + f_1\, \frac{b}{v}\, \dot{q}_1 \int_L \Phi^2{}_1\, dx + f_2\, \frac{b}{2\, v}\, \dot{q}_2 \int_L \Phi_1\, \Phi_2\, dx \right) = 0 \ldots\ldots (43a)$$

and

$$(\ddot{q}_2 + \omega^2{}_2\, q_2) \int \Phi^2{}_2\, dx - \frac{s\, b\, v^2}{2\, m\, r^2} \left(f_1\, q_2 \int_L \Phi^2{}_2\, dx + f_1\, \frac{b}{v}\, \dot{q}_1 \int_L \Phi_1\, \Phi_2\, dx - f_3\, \frac{b}{2\, v}\, \dot{q}_2 \int_L \Phi^2{}_2\, dx \right) = 0 \ldots\ldots (43b)$$

The definite integrals represent constants which can be easily computed from the assumed displacement forms Φ_1 and Φ_2. They reflect the geometric and dynamic properties of the whole structure under consideration. The unknown functions in these two differential equations of second order are q_1 and q_2.

The principal modes Φ_1 and Φ_2 contain arbitrary factors N_1 and N_2 which may be chosen in such a way that

$$\int_L \Phi^2{}_1\, dx = 1 \ldots\ldots (44a)$$

and

$$\int_L \Phi^2{}_2\, dx = 1 \ldots\ldots (44b)$$

The functions Φ are normalized. In using the normalized form of Φ_1 and Φ_2, the substitution factor, D, defined by

$$\int_L \Phi_1 \, \Phi_2 \, dx = D \quad \text{.......................(45)}$$

may be introduced. Eqs. 26, therefore, assumed the more condensed form:

$$\ddot{q}_1 + \omega^2{}_1 \, q_1 + \frac{s \, b \, v^2}{m \, b^2}\left[f_1\left(q_2 \, D + \frac{b}{v}\,\dot{q}_1\right) + f_2 \, \frac{b}{2\,v}\,\dot{q}_2 \, D\right] = 0 \quad \text{....(46a)}$$

and

$$\ddot{q}_2 + \omega^2{}_2 \, q_2 - \frac{s \, b \, v^2}{2 \, m \, r^2}\left[f_1\left(q_2 + \frac{b}{v}\, D \, \dot{q}_1\right) - f_3 \, \frac{b}{2\,v}\,\dot{q}_2\right] = 0 \quad \text{.....(46b)}$$

Eqs. 46 have been derived under the assumption that the structure vibrates in a symmetric configuration. Proceeding in the same manner as previously shown, but presuming an asymmetric oscillation, again leads to the differential equations (Eqs. 46) in which Φ_1, Φ_2, ω_1, and ω_2 now refer to two corresponding asymmetric normal modes. Therefore, Eqs. 46 may be considered the basic system of differential equations on which at least an approximate solution of the flutter problem of suspension bridges can be based.

In the particular case of suspension bridges of orthodox design, Eqs. 46 can be somewhat simplified. It was pointed out previously that in that case the vertical and torsional mode are identical—that is, $\Phi_1 = \Phi_2$. Thus, it follows that the integrals in Eqs. 43 drop out and that these equations reduce to

$$\ddot{q}_1 + \omega^2{}_1 \, q_1 + \frac{s \, b \, v^2}{m \, b^2}\left[f_1\left(q_2 + \frac{b}{v}\,\dot{q}_1\right) + f_2 \, \frac{b}{2\,v}\,\dot{q}_2\right] = 0 \quad \text{.......(47a)}$$

and

$$\ddot{q}_2 + \omega^2{}_2 \, q_2 - \frac{s \, b \, v^2}{2 \, m \, r^2}\left[f_1\left(q_2 + \frac{b}{v}\,\dot{q}_1\right) - f_3 \, \frac{b}{2\,v}\,\dot{q}_2\right] = 0 \quad \text{......(47b)}$$

Eqs. 47 are independent of Φ_1 and Φ_2—that is, of the particular displacement form of the two modes associated with the frequencies ω_1 and ω_2. Eqs. 47 become identical with the equation of motion of a small independent section of the bridge of unit length. Thus, the flutter characteristics—critical velocity and flutter frequency of the whole suspension bridge—are the same as the flutter characteristics of an independent section of the bridge. Therefore, it may be concluded that self-excitation of suspension bridges of that type, where vertical and angular vibrations show the same vibration form, can be studied on section models free to vibrate both vertically and in torsion. The test results, if corrected for the difference in aspect ratio, must coincide with the results of tests on full models.

As the outset of the foregoing discussion concerning the equations of motion, the statement was made that two generalized coordinates suffice to arrive at an exact solution of the flutter problem in the case of suspension bridges of orthodox type (Eqs. 47). To prove this, it may be assumed that

$$\eta = q_1 \, \Phi + q_3 \, \Phi' \quad \text{.....................(48a)}$$

and

$$\phi = q_2 \Phi + q_4 \Phi' \quad \text{(48b)}$$

in which Φ' is a principal mode different from Φ. The four generalized coordinates q lead to four differential equations with four unknowns. However, as a consequence of the orthogonality relation,

$$\int_L \Phi \, \Phi' \, dx = 0 \quad \text{(49)}$$

which exists between two different principal modes of oscillation of a vibrating mechanical system, it will be found that the first two equations involve only the unknowns q_1 and q_2; the third and fourth equations, the unknowns q_3 and q_4. Accordingly, the four equations split up into two independent systems of two equations having the form of Eqs. 47. The group controlled by the lowest frequencies, ω_1 and ω_2, furnishes the lowest critical velocity. The second group leads to a higher value of the critical velocity and is therefore meaningless. From this it is concluded that Eqs. 47 actually constitute the complete system of differential equations involved in the solution of the flutter problem of the orthodox system and the solution, therefore, must be exact.

Effect of Structural Damping.—Thus far the vibration has been considered as not being influenced by any kind of structural damping. It is not difficult, however, to insert in Eqs. 46 or Eqs. 47 an adequate damping term to account for the dissipation of energy due to internal and frictional damping. It is convenient to introduce the internal damping forces as being in phase with the velocity and of a magnitude proportional to the restoring forces. This is tantamount to the assumption that the damping forces are proportional to the amplitude and are independent of the frequency. The restoring forces are represented by the terms $\omega^2{}_1 \, q_1$ and $\omega^2{}_2 \, q_2$, which may be replaced by

$$\omega^2{}_1 \, q_1 \, (1 + i \, g_1) \quad \text{(50a)}$$

and

$$\omega^2{}_2 \, q_2 \, (1 + i \, g_2) \quad \text{(50b)}$$

respectively, in which g_1 and g_2 denote the damping coefficients for vertical and torsional vibrations, respectively,[12] and are related to δ_s, the respective logarithmic decrements of internal damping, by

$$g = \frac{\delta_s}{\pi} \quad \text{(50c)}$$

In so far as the over-all structural damping due to internal and frictional damping forces can be replaced by some equivalent internal damping, characterized by a constant logarithmic decrement δ_s, the coefficients g_1 and g_2 may be understood as related to the over-all damping capacity of the bridge. The effect of structural damping on the flutter characteristics may then be traced by Eqs. 46 or 47 in which $\omega^2{}_1 \, q_1$ and $\omega^2{}_2 \, q_2$ have been replaced by Eqs. 50*a* and

[12] "Mechanism of Flutter, a Theoretical and Experimental Investigation of the Flutter Problem," by T. Theodorsen and I. E. Garrick, *Technical Report No. 685*, National Advisory Committee for Aeronautics, Washington, D. C., 1940.

50*b*, respectively. It should be emphasized that the mathematical expressions for the air forces represent the totality of the damping and exciting air forces acting on the vibrating structure. It would be erroneous to include in the damping decrement the effect of atmospheric damping in still air.

5. Determination of the Flutter Characteristics

In the following mathematical analysis reference is made to the general form of the differential equations of motion (Eqs. 46). The incipient state of flutter has been characterized in Section 2 as a sinusoidal motion of constant amplitude where both components of the coupled motion have the same frequency. Such an oscillation can be represented by the following set of solutions of Eqs. 46:

$$q_1 = u_1\, e^{i \omega t} \qquad (51a)$$

and

$$q_2 = u_2\, e^{i \omega t} \qquad (51b)$$

in which u_1 and u_2 are dimensionless parameters determining the contributions of the vertical and torsional vibrations to the resultant motion. Substitution of Eqs. 51 in Eqs. 46 gives

$$b\left(- \omega^2 + \omega^2_1 + \frac{s\,v}{m} f_1\, i\, \omega \right) u_1 + \left(\frac{s\,v^2}{m} f_1 + \frac{s\,b\,v}{2\,m} f_2\, i\, \omega \right) D\, u_2 = 0 \qquad (52a)$$

and

$$b\left(- \frac{s\,b\,v}{2\,m\,r^2} f_1\, i\, \omega \right) D\, u_1 + \left(- \omega^2 + \omega^2_2 - \frac{s\,b\,v^2}{2\,m\,r^2} f_1 + \frac{s\,b^2\,v}{4\,m\,r^2} f_3\, i\, \omega \right) u_2 = 0 \qquad (52b)$$

Since f_1, f_2, and f_3 are functions of $k = \dfrac{\omega\, b}{v}$, it is convenient to eliminate the wind velocity v by replacing it by $\dfrac{\omega\, b}{k}$. By doing this and dividing both equations by ω^2, Eqs. 52 become

$$b\left(- 1 + \frac{\omega^2_1}{\omega^2} + \frac{s\,b}{m} \frac{f_1}{k}\, i \right) u_1 + \left(\frac{s\,b^2}{m} \frac{f_1}{k^2} + \frac{s\,b^2}{m} \frac{f_2}{2\,k}\, i \right) D\, u_2 = 0 \qquad (53a)$$

and

$$\left(- \frac{s\,b}{m} \frac{b^2}{2\,r^2} \frac{f_1}{k}\, i \right) D\, u_1 + \left(- 1 + \frac{\omega^2_2}{\omega^2_1} \frac{\omega^2_1}{\omega^2} - \frac{s\,b}{m} \frac{b^2}{2\,r^2} \frac{f_1}{k^2} + \frac{s\,b}{m} \frac{b^2}{4\,r^2} \frac{f_3}{k}\, i \right) u_2 = 0 \qquad (53b)$$

Substituting,

$$\mu = \frac{s\,b}{m} \qquad (54a)$$

$$\kappa = \frac{2\,r^2}{b^2} \qquad (54b)$$

and

$$Z = \frac{\omega^2_1}{\omega^2} \qquad (54c)$$

Eqs. 53 may finally be written

$$\left(-1 + Z + \mu \frac{f_1}{k} i\right) u_1 + \mu \left(\frac{f_1}{k^2} + \frac{f_2}{2\,k} i\right) D\, u_2 = 0 \qquad (55a)$$

and

$$-\mu \frac{f_1}{k} i\, D\, u_1 + \left[-\kappa + \kappa \frac{\omega^2_2}{\omega^2_1} Z - \mu \left(\frac{f_1}{k^2} - \frac{f_3}{2\,k} i\right)\right] u_2 = 0 \qquad (55b)$$

which represent two homogeneous linear equations with the variables u_1 and u_2. Finite values of u_1 and u_2 exist only when the coefficient determinant Δ of the system becomes zero. Therefore, $\Delta = 0$ represents the frequency equation from which ω (included in Z) and also—as explained immediately hereafter—the critical velocity v_c, associated with ω, may be determined.

The coeficients of u_1 and u_2 in Eqs. 55 contain imaginary terms. Moreover, the functions f_1, f_2, and f_3 are complex functions of k. The determinant Δ, therefore, is a determinant of second order of complex numbers. Accordingly, the frequency equation or, as it may now be called, the stability condition, takes the form:

$$\Delta = \Delta_1 + i\,\Delta_2 = 0 \qquad (56a)$$

and this condition is satisfied when

$$\Delta_1 = 0 \qquad (56b)$$

and

$$\Delta_2 = 0 \qquad (56c)$$

Thus two equations are obtained from which the two remaining unknowns k and Z, which finally define ω and v_c, may be computed. By substitution of the numerical values of k and Z in either Eq. 55a or Eq. 55b, the ratio $\frac{u_1}{u_2}$ can be determined. In this way flutter speed and flutter mode are found. The ratio $\frac{u_1}{u_2}$ is a complex number—

$$a + b\,i = e^{i\theta} \qquad (57)$$

—and the solutions (Eqs. 51), therefore, appear in the form:

$$q_2 = u_2\, e^{i\omega t} \qquad (58a)$$

and

$$q_1 = u_2\, e^{i\theta}\, e^{i\omega t} = u_2\, e^{i(\omega t + \theta)} \qquad (58b)$$

The symbol θ indicates the difference between the phase angles of the vertical and torsional component of the motion; u_2 is an arbitrary parameter.

From Eqs. 55 the frequency equation (Eq. 56a) can be derived by performing the multiplications. Replacing $\frac{\omega^2_2}{\omega^2_1}$ by X the stability condition—

$$\kappa\, X\, Z^2 - \left[\kappa + \kappa\, X - \mu\, \kappa\, X \frac{f_1}{k} i + \mu \left(\frac{f_1}{k^2} - \frac{f_3}{2\,k} i\right)\right] Z + \left[\kappa - \kappa\, \mu \frac{f_1}{k} i + \mu \left(\frac{f_1}{k^2} - \frac{f_3}{2\,k} i\right) - \mu^2 \frac{f^2_1}{k^3} (1 - D^2)\, i - \mu^2 \frac{f_1}{2\,k^2} (f_2\, D^2 + f_3)\right] = 0 \qquad (59)$$

—is obtained. When the frequencies ω_1 and ω_2 of a suspension bridge are associated with the same displacement form, or in the case of vibration of a section model, Eq. 59 may be simplified by assuming $D = 1$.

For any given design, ω_1, ω_2, κ, and μ are known magnitudes. The two variables Z and k, representing functions of the flutter frequency ω and the critical velocity v_c, remain as unknown quantities. After substituting the numerical values for ω_1, ω_2, κ, and μ in Eq. 59, the real terms can be separated from the imaginary terms. Equating the real part Δ_1 of Eq. 59 and the imaginary part Δ_2, respectively, to zero, two equations with real coefficients are obtained, which finally define Z and k.

The coefficients f_1, f_2, and f_3 are complicated complex functions of k and depend, therefore, on v_c and ω, both unknown quantities at the start of the calculation. It is not possible to solve Eqs. 56b and 56c explicitely for Z and k, and even solution by a process of trial and error would be rather laborious. It is far more convenient to consider k as a parameter and to solve the two equations by treating Z and one of the design factors (for instance, X) as variables. This leads to the following procedure:

A set of consecutive values of k in the neighborhood of the expected value of this magnitude may be chosen and for each of them the terms f_1 and f_2, and f_3 may be calculated using the values of $F(k)$ and $G(k)$ from Table 1. Each value of k and the corresponding value of f_1, f_2, and f_3 are introduced in Eq. 59, which can be split in two equations of the form:

$$X Z^2 + a X Z + b Z + c = 0 \quad (60a)$$

and

$$X Z + dZ + e = 0 \quad (60b)$$

In the case of a damped vibration Eq. 60b takes the form:

$$X Z^2 + dZ^2 + e Z + f = 0 \quad (60c)$$

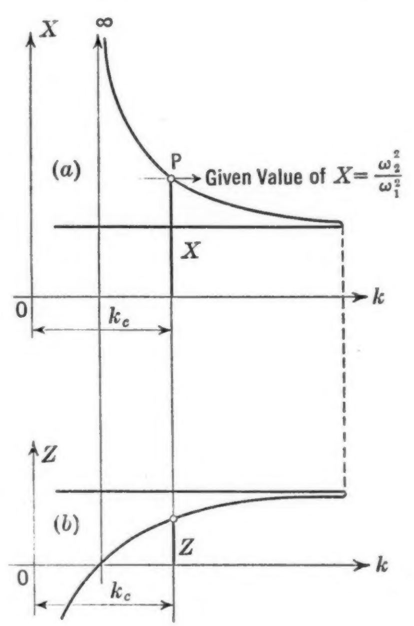


Fig. 7

In Eqs. 60 the coefficients a, b, c, d, e, and f are real numbers. Elimination of $X Z$ yields a quadratic in Z, from which two Z-roots and therefore two X-roots result for each assumed value of k. Plotting Z and X against k gives two curves of the type shown in Fig. 7. That point P of the X-diagram which coincides with the given value of $X = \frac{\omega^2{}_2}{\omega^2{}_1}$ determines the abscissa k_c, the critical value of k, and, simultaneously, the corresponding ordinate in the Z-diagram. Both the values of k_c and Z ultimately define ω and v_c inasmuch as, by defi-

nition,

$$\omega = \frac{\omega_1}{\sqrt{Z}} \qquad (61)$$

and

$$v_c = \frac{\omega\, b}{k_c} \qquad (62)$$

in which ω_1 is the known frequency of the first vertical principal mode of vibration of the system.

Damped Vibration.—In substituting the expressions (Eq. 50) for the terms $\omega^2_1\, q_1$ and $\omega^2_2\, q_2$ in Eqs. 46, the equations of the damped motion are obtained. For the sake of simplicity, it may be assumed that $g_1 = g_2 = g$. Again proceedin in the same manner as before, it can be easily verified that the magnitude Z in Eqs. 55 transforms to $Z(1 + i\, g)$. Accordingly, Eq. 59 assumes the form:

$$\kappa\, X\, Z^2\, (1 + i\, g)^2 - \left[\kappa\, (1 + X) - \kappa\, \mu\, X \frac{f_1}{k}\, i + \mu \left(\frac{f_1}{k^2} - \frac{f_3}{2\,k}\, i \right) \right] Z\, (1 + i\, g)$$
$$+ \left[\kappa - \kappa\, \mu\, \frac{f_1}{k}\, i + \mu \left(\frac{f_1}{k^2} - \frac{f_3}{2\,k}\, i \right) - \mu^2 \frac{f^2_1}{k^3}\, (1 - D^2)\, i \right.$$
$$\left. - \mu^2 \frac{f_1}{2\,k^2}\, (f_2\, D^2 + f_3) \right] = 0 \qquad (63)$$

Since g is small compared with unity, the term $(1 + i\, g)^2$ conveniently may be replaced by $(1 + 2\, i\, g)$. As for the remainder, the procedure of solving Eq. 63 is the same as discussed previously.

Illustrative Problem.—The values underlying the following calculation coincide with the design data of the prototype of one of the full models of the Tacoma Narrows Bridge (preliminary design with stiffening trusses and closed deck) which have been investigated in the wind tunnel of the Structural Research Laboratory of the University of Washington. The behavior of that model under wind action so clearly indicated a manifestation of flutter that the design of the prototype may be chosen to demonstrate the application of the method outlined in Section 5 as an example of self-excitation by pure flutter. Because of the favorable shape of the leading edge, the dynamic behavior of the model apparently was influenced only to a minor degree by vortex shedding at that point. Consequently, the suspended floor structure may be considered as having flat plate characteristics.

The frequencies of the first asymmetric vertical and torsional modes of the bridge system under consideration are considerably higher than the frequencies of the corresponding symmetric natural modes. Accordingly, it must be expected that self-excitation under wind action is associated with a symmetric configuration of the vibrating system. The investigation, therefore, must start from frequencies and displacement forms of the symmetric modes of vibration.

As a consequence of the considerable torsional stiffness of the towers of the model, the shape of the first torsional mode deviates appreciably from the shape of the corresponding vertical mode. The two configurations are shown in Fig. 8.

Design data for the prototype include: Length of main span, $L_m = 2{,}800$ ft; length of side span, $L_s = 1{,}100$ ft; width of bridge, $2\,b = 60$ ft; mass of bridge, $m = 269$ slugs per lin ft; and mass radius of gyration, $r = 23.4$ ft. Frequencies and displacement forms of the first vertical and the first torsional principal modes have been calculated by the Ritz method, discussed in Section 4, using a two-term sine expansion for the main span and a single sine function for the side spans. The calculated and observed frequencies show close agreement. For the first vertical mode the frequency is

$$\omega^2_1 = 0.755 \quad \ldots\ldots(64a)$$

and the displacement forms for the main span and the side span, respectively, are

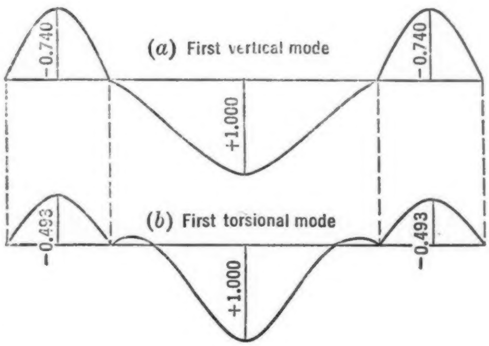


FIG. 8

$$\Phi_1 = 0.792 \left(\sin \frac{\pi\, x}{L_m} - 0.113 \sin \frac{3\, \pi\, x}{L_m} \right) \quad \ldots\ldots\ldots\ldots(64b)$$

and

$$\Phi_1 = 0.792 \left(0.740 \sin \frac{\pi\, x}{L_s} \right) \quad \ldots\ldots\ldots\ldots\ldots\ldots(64c)$$

For the first torsional mode the frequency is

$$\omega^2_2 = 2.41 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(65a)$$

and the displacement forms for the main span and the side span, respectively, are

$$\Phi_2 = 0.835 \left(\sin \frac{\pi\, x}{L_m} - 0.493 \sin \frac{3\, \pi\, x}{L_m} \right) \quad \ldots\ldots\ldots\ldots(65b)$$

and

$$\Phi_2 = 0.835 \left(0.493 \sin \frac{\pi\, x}{L_s} \right) \quad \ldots\ldots\ldots\ldots\ldots\ldots(65c)$$

The functions Φ_1 and Φ_2 are given in normalized form, the factors of normalization being $N_1 = 0.792$ and $N_2 = 0.835$. According to Eq. 45, therefore,

$$D = \int_L \Phi_1\, \Phi_2\, dx = 0.792 \times 0.835 \left[\int_0^{L_m} \left(\sin \frac{\pi\, x}{L_m} - 0.113 \sin \frac{3\, \pi\, x}{L_m} \right) \right.$$
$$\left(\sin \frac{\pi\, x}{L_m} - 0.493 \sin \frac{3\, \pi\, x}{L_m} \right) dx + 2 \int_0^{L_s} \left(0.740 \sin \frac{\pi\, x}{L_s} \right.$$
$$\left. \left. \times\, 0.493 \sin \frac{\pi\, x}{L_s} \right) dx \right] \quad \ldots\ldots\ldots\ldots\ldots\ldots(66)$$

from which $D = 0.888$ and $D^2 = 0.788$. Using $s = 2\,\rho\,\pi\,b = 6.28 \times 0.00238 \times 30 = 0.448$, Eq. 54*a* gives $\mu = \dfrac{0.448 \times 30}{269} = 0.05$. From Eq. 54*b* it follows that $\kappa = \dfrac{2 \times 560}{30^2} = 1.25$; $X = \dfrac{\omega^2_2}{\omega^2_1} = \dfrac{2.41}{0.755} = 3.19$.

After these preparatory calculations the stability condition (Eq. 59) may be applied to a set of assumed values of k to obtain the two diagrams shown in Fig. 7 or, at least, those parts of the two graphs involving the region where the roots X and Z of Eq. 59 may be expected to be found. Assuming that the value of k lies between 0.20 and 0.25, the determination of three values of Z and the corresponding three values of X suffices to determine k_c with sufficient accuracy. The stability condition (Eq. 59) has been solved, therefore, for k-values of 0.20, 0.22, and 0.25. The computation based on $k = 0.20$ is as follows:

From Table 1, $F(k) = 0.728$ and $G(k) = 0.189$. Therefore, $f_1 = 0.728 - 0.189\,i$; $f_2 = 1 + f_1 = 1.728 - 0.189\,i$; and $f_3 = 1 - f_1 = 0.272 + 0.189\,i$. According to Eq. 59,

$$\kappa\,X\,Z^2 = 1.25\,X\,Z^2 \quad \text{.........(67a)}$$

$$- \left[\kappa\,(1 + X) - \kappa\,\mu\,X\frac{f_1}{k}\,i + \mu\left(\frac{f_1}{k^2} - \frac{f_3}{2\,k}\,i\right)\right] Z$$

$$= -\,1.1909\,X\,Z - 2.1842\,Z + 0.2275\,i\,X\,Z + 0.2703\,i\,Z \quad \text{....(67b)}$$

and

$$\kappa + \mu\left(\frac{f_1}{k^2} - \frac{f_3}{2\,k}\,i\right) - \kappa\,\mu\frac{f_1}{k}\,i - \mu^2\left[\frac{f^2_1}{k^3}\,i\,(1 - D^2)\right.$$

$$\left. + \frac{f_1}{2\,k^2}\,(f_2\,D^2 + f_3)\right] = 2.0696 - 0.5215\,i \quad \text{.........(67c)}$$

The sum of Eqs. 67 represents Eq. 59; the real terms furnishing the equation,

$$1.25\,X\,Z^2 - 1.1909\,X\,Z - 2.1842\,Z + 2.0696 = 0 \quad \text{........(68a)}$$

and the imaginary terms furnishing the equation,

$$0.2275\,X\,Z + 0.2703\,Z - 0.5215 = 0 \quad \text{.........(68b)}$$

From Eq. 68*b*,

$$X\,Z = -\,1.1881\,Z + 2.2923 \quad \text{.........(68c)}$$

which, when substituted in Eq. 68*a*, gives the quadratic equation,

$$Z^2 - 1.4114\,Z - 0.6603 = 0 \quad \text{.........(68d)}$$

having the roots: $Z_1 = 0.475$ and $Z_2 = 0.937$. Substituting these values in Eq. 68*c* gives $X_1 = -\,1.1881 + \dfrac{2.292}{0.475} = 3.64$ and $X_2 = -\,1.1881 + \dfrac{2.292}{0.937} = 1.26$.

In the same manner, two other pairs of the roots X and Z have been determined for $k = 0.22$ and $k = 0.25$. The results are compiled in Table 2.

Since the values of the roots X_2, representing the lower branch of the X-curve, are far below the given value of $X = \dfrac{\omega^2_2}{\omega^2_1} = 3.19$, these values and

TABLE 2.—VALUES OF X AND Z IN EQ. 59 CORRESPONDING TO ASSUMED VALUES OF k IN ILLUSTRATIVE PROBLEM

k (assumed) (1)	X_1 (2)	X_2 (3)	Z_1 (4)	Z_2 (5)
0.200	3.64	1.26	0.475	0.937
0.220	2.87	1.24	0.552	0.938
0.250	2.31	1.18	0.620	0.939

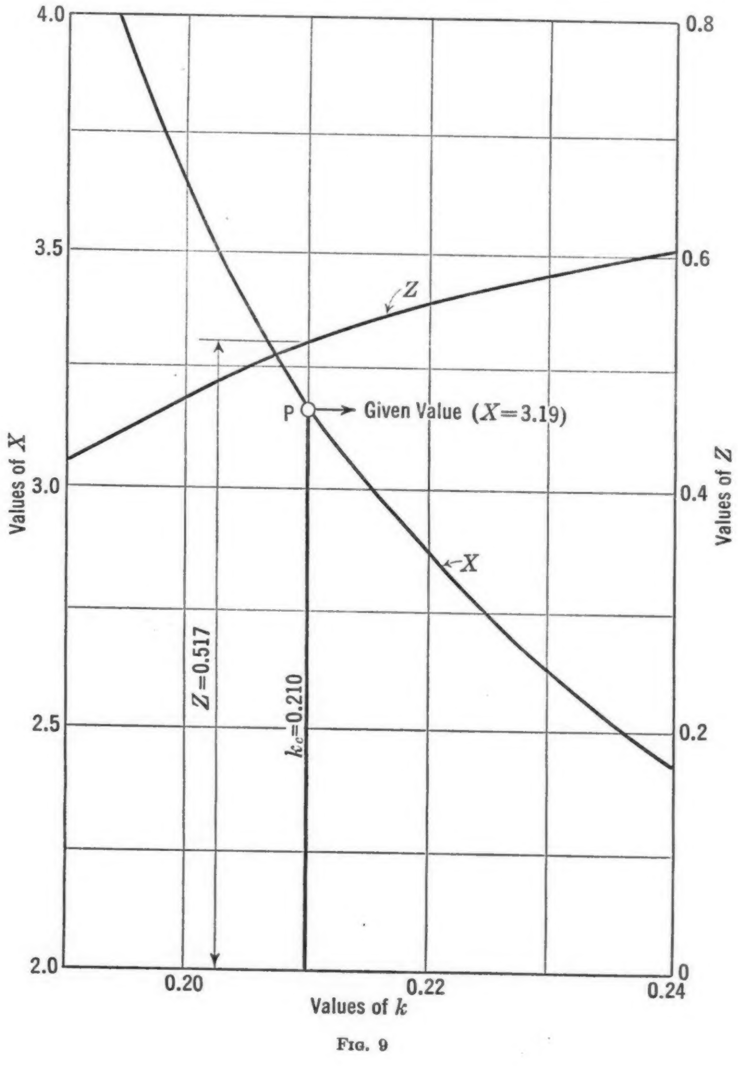


FIG. 9

therefore the corresponding values of Z_2 are of no significance for the solution. The curves X_1 and Z_1 are shown in Fig. 9. A parallel to the k-axis at the distance $X = 3.19$ from it intersects the X-diagram at P. The corresponding abscissa, $k_c = 0.210$, represents the critical value of k. From the Z-diagram the value of $Z = 0.517$, which corresponds to the abscissa $k = 0.210$, is inferred. These values, when used in Eqs. 61 and 62, give the flutter frequency $\omega = \dfrac{0.869}{\sqrt{0.517}} = 1.21$ or 11.6 cycles per min; and the critical velocity $v_c = \dfrac{1.21 \times 30}{0.210} = 173$ ft per sec, or 118 miles per hr. The flutter frequency lies between $\omega_1 = 0.869$ and $\omega_2 = 1.553$, the frequencies of the principal modes.

In order to obtain information about the particular character of the excited motion reference is made to Eq. 55*a*, from which

$$\frac{u_1}{u_2} = -\frac{\mu\left(\dfrac{f_1}{k^2} + \dfrac{f_2}{2\,k}\,i\right) D}{-1 + Z + \mu\dfrac{f_1}{k}\,i} \qquad (69)$$

Substituting the numerical values of f_1 and f_2 from Table 1 for the value of $k = 0.210$ ($f_1 = 0.720 - 0.189\,i$ and $f_2 = 1.720 - 0.189\,i$), $-\mu\left(\dfrac{f_1}{k^2} + \dfrac{f_2}{2\,k}\,i\right) D = -0.747 + 0.0085\,i$ and $-1 + Z + \mu\dfrac{f_1}{k}\,i = -0.438 + 0.1714\,i$. Using vector representation these values may be written: $-0.747 + 0.0085\,i = 0.747\,e^{i\theta_1}$ and $-0.438 + 0.1714\,i = 0.470\,e^{i\theta_2}$, from which $\tan\theta_1 = -0.011$ and $\tan\theta_2 = -0.391$, so that $\theta_1 = -0.011$ and $\theta_2 = -0.372$. Therefore, $\dfrac{u_1}{u_2} = \dfrac{0.747\,e^{-0.011i}}{0.470\,e^{-0.372i}} = 1.59\,e^{0.36i}$, which means that the ratio of the vertical and the torsional amplitudes becomes 1.59 and the difference of the phase angles is 0.36 radian, or 20.5°. The motion, therefore, is described by the equations:

$$\phi = C \sin \omega\, t \qquad (70a)$$

and

$$\eta = 1.59\, C \sin(\omega\, t + 0.36) \qquad (70b)$$

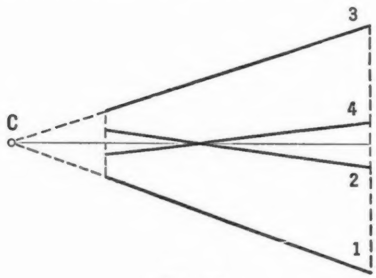


FIG. 10

Fig. 10 shows successive positions of the bridge floor during one cycle. The extreme position of the apparent center of rotation, indicated by point C, the intersection of the two lines 1 and 3 (extreme positions) lies upstream ahead of the leading edge. Lines 2 and 4 indicate the positions when η approaches zero.

To check the effect of damping on the flutter characteristic the procedure was repeated by using Eq. 63, assuming $g = 0.016$, which value corresponds with a logarithmic decrement of structural damping, $\delta = 0.05$, which in turn

agrees fairly well with the observed damping rate of the model of the Tacoma Narrows Bridge.

The computed X-curve and the Z-curve are shown in Fig. 11. Point P, having the ordinate $X = 3.19$, defines the abscissa $k_c = 0.195$ and the corre-

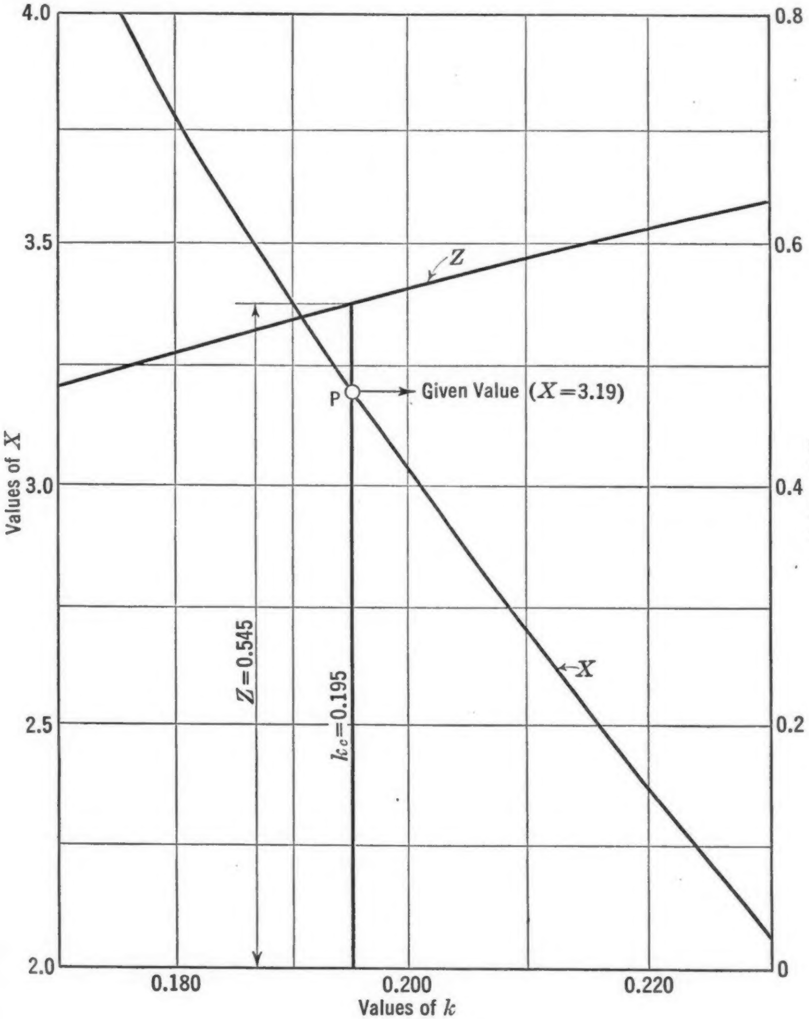


FIG. 11

sponding ordinate $Z = 0.545$. Using these values, $\omega = \dfrac{0.869}{\sqrt{0.545}} = 1.18$, or 11.3 cycles per min; and $v_c = \dfrac{1.18 \times 30}{0.195} = 182$ ft per sec, or 124 miles per hr. The flutter frequency of the damped motion is slightly less than that of the undamped motion. The damping effect increases the critical velocity about 6%.

No precise data concerning observations on the coupled motion of the full model of the Tacoma Narrows Bridge are available. The purpose of the test was achieved after studying the behavior of the model under various angles of attack. Within the limits of wind velocity available in the wind tunnel (118 miles per hr), no self-excited vibration could be observed at zero angle of attack. However, by a process of extrapolation a probable value of v_c at zero angle of attack could be disclosed. The curve in Fig. 12 is taken from a report of the Structural Research Laboratory of the University of Washington showing the critical wind velocities plotted against angle of attack. In projecting the curve toward $\alpha = 0°$ as indicated by the dashed part of the curve, a probable value of v_c at $\alpha = 0°$ can be estimated, inasmuch as the curve intersects the vertical axis somewhere between values of 120 miles per hr and 130 miles per hr. The previously derived theoretical value of 124 miles per hr is in fair agreement with any value between these two limits.

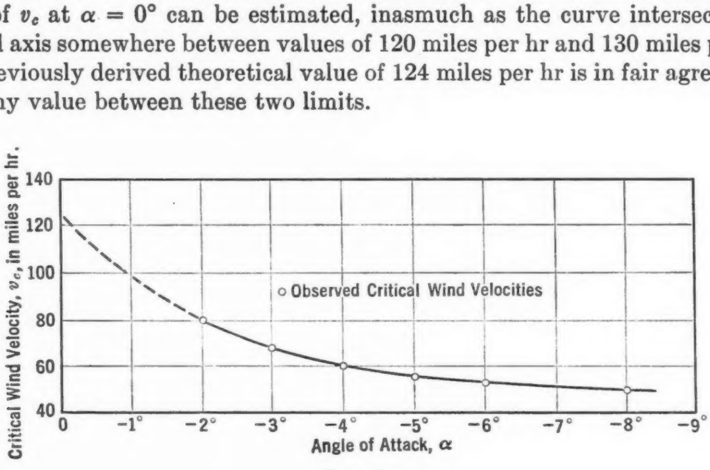


FIG. 12

The self-excited motion observed (under angles of attack of −2° and more) showed the characteristic feature of the flutter phenomenon—a torsional motion superimposed on a vertical oscillation. The apparent center of rotation was offset toward the windward truss as predicted by the theory. The distance of the center of rotation from the longitudinal axis of the structure changed with the changing angle of the acting wind. A preliminary theoretical investigation of the effect of varying angles of attack on the critical wind velocity and the flutter frequency shows that the torsional component of the coupled motion becomes more and more dominant with steadily increasing angle of attack. Accordingly, the distance of the center of rotation from the center line of the structure diminishes when the angle of attack increases. Likewise, the flutter frequency increases and approaches the frequency ω_2 of the torsional mode when α steadily increases.

It is interesting to check the behavior under wind action of a section model having the same dynamic properties as the full model. The fact that the two natural modes composing the excited motion are distinctly different in their displacement form renders it probable that ω and v_c might have somewhat different values for the two kinds of models—which is actually the case. Application of Eq. 59 (using $D = 1$) yields, when the values are adjusted to

prototype scale, $\omega = 1.23$ and $v_c = 114$ miles per hr, as compared with $\omega = 1.18$ and $v_c = 124$ miles per hr for the complete bridge, indicating a noticeable difference between the critical velocities of the full model and the section model. The same structural damping rate as introduced in the calculations for the full model has been taken into account.

6. Generalized Theory of Dynamic Stability of Truss-Stiffened Suspension Bridges

Extension of the Theory of Flutter.—In the preceding sections concern was centered on the specification of the stability condition of suspension bridges having an ideal flat plate cross section. The stability condition has been derived under the assumption that the dynamic air forces responsible for the occurrence of self-excitation are defined by the Theodorsen equation for a perfectly streamlined thin flat plate. It is easily noted that this greatly simplified concept—presumably in the majority of cases—might be not quite consistent with the actual conditions regarding the magnitude and distribution of the air forces.

It is felt, therefore, that the theory developed in the preceding sections needs further refinement, taking into account the more or less marked effect of the particular shape of the cross section in the vicinity of the leading edge. The blunt form of the leading edge gives rise to a periodic lift force F_v, as pointed out in Section 1, which may be considered as simply superimposed upon the air forces acting on the flat plate (Fig. 1).

The subsequent theory is based on three assumptions, the first of which is that the air forces acting on the flat plate are defined by the Theodorsen equations (Eqs. 7 and 8). It is readily realized that the lift force F_v may have some influence on the air flow around the structure; but it can be assumed that, when F_v is of magnitude comparable to that of the flat plate forces, magnitude and distribution of those forces will not be influenced substantially. Comparison of the prediction of the theory with test results indicates that the assumption is fairly correct. The second assumption is that the frequency of the vortex shedding at the windward chord—the vortices assumed to be responsible for the alternating force F_v—is controlled by the frequency of the oscillating structure; the third, that the lift force F_v is assumed to be a function of the wind velocity v, the torsional amplitude ϕ, and the first derivative $\dot{\eta}$ of the vertical amplitude and that this function has the form:

$$F_v = 2\,\pi\,\rho\,b\,v^2\,(A_v + B_v\,i\,)\left(\phi + \frac{b\,\dot{\eta}}{v}\right) \ldots\ldots\ldots\ldots (71)$$

in which A_v and B_v are parameters depending on the shape of the cross section, are supposed to be determined by tests on a section model of the design under consideration, and are functions of k presumably slightly decreasing with increasing k. Eq. 71 may be regarded as a hypothesis whose validity and limitation can be proved only by a number of laboratory tests. No attempt is made to establish in a rational way the mathematical form of this equation.

The following analysis is confined to cases—apparently including suspension bridges of the truss type—where the vortex shedding effect is comparatively

small, about of the order of magnitude of the damping effect of the Theodorsen air forces. The theory does not apply to girder-stiffened bridges, where F_v becomes dominant and the air forces acting on the plate presumably deviate considerably in magnitude and distribution from those given by the Theodorsen equations.

By introducing the substitution factors, s, and

$$f_4 = A_v + B_v\, i \quad \ldots\ldots (72)$$

Eq. 71 becomes

$$F_v = s\, v^2 f_4 \left(\phi + \frac{b\,\dot{\eta}}{v}\right) \quad \ldots\ldots (73)$$

which is a more suitable form for the analysis. The lift force F_v produces a moment $b\, F_v$ about the longitudinal center line of the structure. Adding F_v and $b\, F_v$ to the flat plate forces as represented by Eqs. 7 and 8, the expressions for the resultant lift force F_L and the resultant moment M_L which operate to produce self-excitation now assume the form:

$$F_L = -\, s\, v^2 \left[f_1 \left(\phi + \frac{b}{v}\,\dot{\eta}\right) + f_2\,\frac{b}{2\,v}\,\dot{\phi} + f_4 \left(\phi + \frac{b}{v}\,\dot{\eta}\right)\right] \quad \ldots (74)$$

$$M_L = \frac{s\, b\, v^2}{2} \left[f_1 \left(\phi + \frac{b}{v}\,\dot{\eta}\right) - f_3\,\frac{b}{2\,v}\,\dot{\phi} + 2\, f_4 \left(\phi + \frac{b}{v}\,\dot{\eta}\right)\right] \quad \ldots (75)$$

Proceeding in the same manner as in Section 4, under the heading, "Application of Lagrange's Equations to the Flutter Problem of Suspension Bridges," Eqs. 46 become

$$\ddot{q}_1 + \omega^2{}_1\, q_1 + \frac{s\, b\, v^2}{m\, b^2} \left[f_1 \left(q_2\, D + \frac{b}{v}\,\dot{q}_1 \right) + \frac{b\, f_2}{2\, v}\,\dot{q}_2\, D + f_4 \left(q_2\, D + \frac{b}{v}\,\dot{q}_1 \right)\right] = 0 \quad \ldots\ldots (76a)$$

and

$$\ddot{q}_2 + \omega^2{}_2\, q_2 - \frac{s\, b\, v^2}{2\, m\, r^2} \left[f_1 \left(q_2 + \frac{b}{v}\, D\,\dot{q}_1 \right) - \frac{b\, f_3}{2\, v}\,\dot{q}_2 + 2\, f_4 \left(q_2 + \frac{b}{v}\, D\,\dot{q}_1 \right)\right] = 0 \quad \ldots\ldots (76b)$$

Since the first and third terms in Eqs. 74 and 75 show the same factor, $\phi + \frac{b\,\dot{\eta}}{v}$, it is easily noted that the third terms within the brackets in Eqs. 76 are derived from the first terms simply by replacing f_1 with f_4 and $2\, f_4$, respectively. Eqs. 55, therefore, become

$$\left(-1 + Z + \mu\,\frac{f_1}{k}\, i + \mu\,\frac{f_4}{k}\, i \right) u_1 + \mu \left(\frac{f_1}{k^2} + \frac{f_2}{2\, k}\, i + \frac{f_4}{k^2} \right) D\, u_2 = 0 \quad \ldots (77a)$$

and

$$-\left(\mu\,\frac{f_1}{k}\, i + 2\,\mu\,\frac{f_4}{k}\, i \right) D\, u_1 + \left[-\kappa + \kappa\,\frac{\omega^2{}_2}{\omega^2{}_1}\, Z - \mu \left(\frac{f_1}{k^2} - \frac{f_3}{2\, k}\, i \right) - 2\,\mu\,\frac{f_4}{k^2} \right] u_2 = 0 \quad \ldots\ldots (77b)$$

If the determinant of Eqs. 55 is written in condensed form—

$$\Delta_o = \begin{vmatrix} A & B \\ C & E \end{vmatrix} \qquad (78)$$

—in which A, B, C, and E are the coefficients of the variables u_1 and u_2 in Eqs. 55—it is readily noted that the determinant of Eqs. 77 can be written

$$\Delta = \begin{vmatrix} A + \mu \frac{f_4}{k} i & B + \mu \frac{f_4}{k^2} D \\ -\left(C + 2\,\mu \frac{f_4}{k} i\, D\right) & E - 2\,\mu \frac{f_4}{k^2} \end{vmatrix} \qquad (79)$$

Therefore, the stability condition, $\Delta = 0$, assumes the form:

$$\Delta = \Delta_o + \mu f_4 \left(- \frac{2\,A}{k^2} + \frac{E}{k} i + \frac{2\,B}{k} i\, D + \frac{C}{k^2} D \right) \qquad (80)$$

in which the terms containing f^2_4 have been disregarded as small magnitudes of second order. Their sum vanished when $D = 1$. The symbol Δ_o is the determinant $A\,E - B\,C$ and is therefore identical with the left side of Eq. 59. Without introducing appreciable error Eq. 80 may be further simplified by setting

$$A = -1 + Z \qquad (81a)$$

$$B = 0 \qquad (81b)$$

$$C = 0 \qquad (81c)$$

and

$$E = -\kappa + \kappa\, Z\, X \qquad (81d)$$

and by neglecting also all terms containing the product of f_4 and one of the functions f_1, f_2, and f_3. Thus, the expression is finally obtained for the stability condition:

$$\Delta = \Delta_o + \mu f_4 \left[\frac{2}{k^2} (1 - Z) + \frac{\kappa}{k} (-1 + Z\, X)\, i \right] \qquad (82)$$

When structural damping is to be taken into account, Δ_o becomes identical with the left side of Eq. 63.

Eq. 82 departs from the stability condition (Eq. 59) by the two additional terms which depend on f_4 and which represent the corrective effect of the particular shape of the cross section. To specify the condition of instability the same procedure may be used as in Section 5. Eq. 82 can be split into a real part and an imaginary part, thus yielding two equations that ultimately determine critical velocity and flutter frequency.

Dynamic instability of the type of structure herein considered under action of a horizontal wind flow is the combined effect of the flat plate air forces and the lift force F_v. It remains in its essential features a flutter phenomenon—that is, an excited motion composed of a vertical and a torsional vibration. However, the critical velocity and the flutter frequency now are controlled by

both the dynamic properties of the system and the lift force F_v, the latter depending on the particular shape of the cross section. With increasing magnitude of F_v, the critical velocity v_c decreases, whereas the flutter frequency ω increases, tending to approach the frequency of the natural torsional mode which forms one of the two components of the excited motion.

Instability of an Oscillating System with One Degree of Freedom.—The combined effect of the flat plate air forces and the lift force F_v changes the most characteristic feature of the flutter phenomenon. As long as the wind flow produces only the aerodynamic forces as described by the Theodorsen equations, dynamic instability can develop only in a vibrating system which has at least two degrees of freedom. Thus, in the absence of air forces of the type represented by the lift force F_v, the flat plate air forces alone cannot cause any kind of dynamic instability in those cases where the system, due to its particular restraint, oscillates purely vertically or purely torsionally.

This fact can readily be shown: Eq. 47*a* represents, on dropping the terms which depend on the torsional coordinate q_2, the equation of pure vertical motion—

$$m\,\ddot{q}_1 + m\,\omega^2{}_1\,q_1 + s\,v\,f_1\,\dot{q}_1 = 0 \qquad (83a)$$

Inasmuch as

$$f_1 = F - G\,i \qquad (83b)$$

Eq. 83*a* becomes

$$m\,\ddot{q}_1 + m\,\omega^2{}_1\,q_1 + F\,s\,v\,\dot{q}_1 - G\,s\,v\,\dot{q}_1\,i = 0 \qquad (83c)$$

The last term in Eq. 83*c* represents a force in phase with the amplitude q_1. Its sole effect is a slight change of the spring constant $m\,\omega^2{}_1$, it has no bearing at all upon the energy balance of the vibrating system. The preceding term is in phase with the velocity q_1 and represents, because of its plus sign, a damping force which dissipates the energy of the oscillating system. Eq. 83*c* obviously defines a damped vibration.

From Eq. 47*b*, by omitting the terms having the factor q_1, the equation of pure angular vibration—

$$m\,r^2\,\ddot{q}_2 + m\,r^2\,\omega^2{}_2\,q_2 - \frac{s\,b\,v^2}{2} f_1\,q_2 + \frac{s\,b^2\,v}{4} f_3\,\dot{q}_2 = 0 \qquad (84)$$

—is derived. Substituting Eq. 83*b* and

$$f_3 = 1 - F + G\,i \qquad (85)$$

in Eq. 84 gives

$$m\,r^2\,\ddot{q}_2 + m\,r^2\,\omega^2{}_2\,q_2 - \frac{s\,b\,v}{2}\left(F\,v\,q_2 - \frac{G\,b}{2}\,i\,\dot{q}_2\right)$$
$$+ \frac{s\,b\,v}{2}\left[G\,v\,i\,q_2 + (1 - F)\,\frac{b}{2}\,\dot{q}_2\right] = 0 \qquad (86)$$

The third term in Eq. 86 represents a force in phase with the amplitude q_2 which affects only the spring constant. The fourth term always is positive, as G and $(1 - F)$ are real positive numbers. The product $i\,q_2$ is in phase with the velocity $\dot{q}_2$, accordingly, the last term is a dissipative force and Eq. 86 defines a

damped vibration, regardless of the value of the wind velocity v. The foregoing discussion demonstrates that no dynamic instability occurs when an ideal flat plate vibrates in a purely torsional or purely vertical mode.

Now let it be assumed that the complex factor f_4 in the expression for the lift force F_v be so defined that F_v represents an exciting force increasing with increasing wind velocity. Acting on a section model with fixed center of rotation, for instance, the force F_v, at a certain wind velocity at which structural damping is overcome, would generate excited vibrations. However, the flat plate air forces, which are thought of as acting simultaneously on the model, are damping forces, as demonstrated previously, and delay the excitation until a wind velocity is reached at which equilibrium exists between the exciting and damping forces. Any further increase of v causes vibrations of steadily increasing amplitude, and the system becomes dynamically unstable. The wind velocity at which steady state harmonic motion can be maintained defines the critical velocity of the section model under wind action.

Thus, another type of self-excitation is disclosed whereby periodic vortex discharge at the leading edge is the real source of instability. The air forces that act on the flat plate play the role of damping forces with the effect of delaying the start of the self-excited vibration. The frequency of the excited vibration is reduced as compared with the frequency of the motion in still air.

A particular question arises as to the correlation of the behavior under wind action of a system of one degree of freedom, previously discussed, and the behavior of a system of two degrees of freedom treated in this section (see "Extension of the Theory of Flutter"). In a system having two degrees of freedom, the flutter phenomenon with its peculiar mechanism and the vortex shedding effect operate together to produce the dynamic instability of the structure, whereas in a system whose motion is restricted to torsional oscillations, only F_v is responsible for the occurrence of excited vibrations. Thus, it follows that two identical section models, one of them spring suspended and the other with fixed center of rotation, will show different values of v_c.

The detailed discussion of the particular behavior of an oscillating system with fixed center of rotation was deemed necessary because of the importance of that class of mechanical system for the determination of those experimental data from which magnitude and angle of phase of F_v can be derived. As only one free coordinate is involved, the theory of a section model whose motion under wind action is restricted to angular vibration becomes very simple.

The equation of motion of a torsionally vibrating section model acted on by the flat plate air forces and by the lift force F_v can be easily derived from Eq. 76*b* by omitting the term containing the coordinate q_1. Thus,

$$\ddot{q}_2 + \omega^2{}_2\, q_2 - \frac{s\,b\,v^2}{2\,m\,r^2}\left(f_1\, q_2 - \frac{b\,f_3}{2\,v}\,\dot{q}_2 + 2\,f_4\,q_2\right) = 0 \quad \ldots\ldots\ldots (87)$$

Substituting the solution $q_2 = e^{i\omega t}$, replacing v with $\frac{\omega\, b}{k}$, Eq. 87 becomes

$$-\,\omega^2 + \omega^2{}_2 - \frac{s\,b}{m}\left(\frac{b^2}{2\,r^2}\frac{f_1}{k^2}\,\omega^2 - \frac{b^2}{4\,r^2}\frac{f_3}{k}\,i\,\omega^2 + 2\,\frac{b^2}{2\,r^2}\frac{f_4}{k^2}\,\omega^2\right) = 0 \quad \ldots (88)$$

Dividing by ω^2 and substituting Eqs. 54 gives

$$-1 + \frac{\omega^2_2}{\omega^2} - \frac{\mu}{\kappa}\left(\frac{f_1}{k^2} - \frac{f_3}{2\,k}\,i\right) - 2\,\frac{\mu}{\kappa}\,\frac{f_4}{k^2} = 0 \qquad (89)$$

which represents the stability condition. In order to include structural damping also, the term $\frac{\omega^2_2}{\omega^2}$ is extended by the factor $(1 + i\,g)$, so that

$$-1 + \frac{\omega^2_2}{\omega^2}(1 + i\,g) - \frac{\mu}{\kappa}\left(\frac{f_1}{k^2} - \frac{f_3}{2\,k}\,i\right) - 2\,\frac{\mu}{\kappa}\,\frac{f_4}{k^2} = 0 \qquad (90)$$

Eq. 90 represents a relationship between the frequency ω of the induced vibration and the magnitude $k = \frac{\omega\,b}{v}$. Because f_1, f_3, and f_4 are complex functions, Eq. 90, on equating the real and the imaginary parts to zero, yields two equations with real coefficients. These equations define ω and k if A_v and B_v are considered known properties of the model setup. In turn, Eq. 90 determines the characteristics A_v and B_v when ω and k would be known through laboratory tests on a section model by observing the frequency ω of the excited motion and the critical velocity v_c.

Eq. 90 forms the basic relation for the interpretation of a group of tests on a section model with fixed center of rotation, which may furnish A_v and B_v, determining the magnitude and the angle of phase of F_v as a function of k for the cross section under consideration. Introduction of these two experimental values into the stability conditions (Eq. 82) permits prediction of flutter frequency and critical wind velocity for any suspension bridge having a cross section of the same geometric form as the section model.

If A_v and B_v were constants—that is, were independent of k—a single test would yield the values of A_v and B_v, because two relations exist between A_v and B_v on the one hand and the observed values of ω and k on the other hand. To find any variation of A_v and B_v at least two or, if necessary, three experiments with different spring constants must be carried out. In this way, two or three values of A_v and B_v can be found which determine sufficiently accurately the A_v-diagram and the B_v-diagram indicating the variation of these two magnitudes with k (Fig. 13).

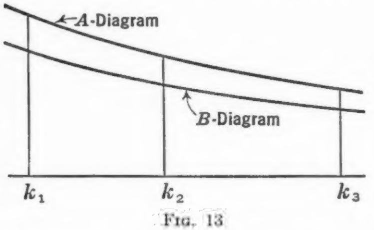


FIG. 13

It is of interest to note that in the case of purely torsional vibration the frequency ω of the excited oscillation lies, according to the theory, definitely below the frequency ω_2 of the free torsional vibration (motion in still air). This shift in frequency actually has been observed in purely torsional motion of section models of truss type.

Illustrative Problem.—Experiments at the University of Washington with section models of the Golden Gate Bridge afford a welcome opportunity to explain the previously outlined method by an instructive example and to show

that the observed data comply fairly well with the predictions of the theory. It will be indicated subsequently: (1) How the air force function f_4 can be found from the observed frequencies and critical velocities of a section model with fixed center of rotation (model A); and (2) how to determine the condition of stability of another section model with free spring mount (coupled motion) having the same geometric form as model A (model B). Using the function f_4 derived from the tests on model A, the stability condition (Eq. 82) applied to model B must yield results which are in accordance with the measurements made on that model.

Section Model with Fixed Center of Rotation (Model A).—The design characteristics of model A, after adjustment to prototype scale are: $2\,b = 90$ ft; $m = 555$ slugs; and $r = 27.8$ ft. In the first test, observed values were as follows: Natural torsional frequency (wind off), $\omega_2 = 0.929$; frequency of the excited torsional motion, $\omega = 0.856$; and critical wind velocity, $v_c = 94$ ft per sec. The frequencies $\omega_2 = 0.929$ and $\omega = 0.856$ correspond, respectively, to 76.8 cycles per min and 70.8 cycles per min observed on the 75-scale model. The logarithmic decrement of structural damping of the model was $\delta_s = 0.073$, which value was determined by subtracting the observed logarithmic decrement of aerial damping, $\delta_a = 0.010$, from the total damping, $\delta_s + \delta_a = 0.083$.

The following values are computed first: $s = 2\,\pi\,\rho\,b = 6.28 \times 0.00238 \times 45 = 0.673$; $\mu = \frac{s\,b}{m} = \frac{0.673 \times 45}{555} = 0.0545$; $\kappa = \frac{2\,r^2}{b^2} = \frac{2 \times 27.8^2}{45^2} = 0.763$; $\frac{\mu}{\kappa} = 0.0715$; $\frac{\omega^2{}_2}{\omega^2} = \frac{0.929^2}{0.856^2} = 1.178$; $k = \frac{\omega\,b}{v_c} = \frac{0.856 \times 45}{94} = 0.410$; and $g = \frac{\delta_s}{\pi} = 0.0232$. Entering Table 1 with the argument $k = 0.41$, it is found that $f_1 = 0.622 - 0.163\,i$ and $f_3 = 0.378 + 0.163\,i$.

Introducing the numerical values of $\frac{\mu}{\kappa}$, $\frac{\omega^2{}_2}{\omega^2}$, k, and g in the stability conditions (Eq. 90) gives

$$- 1 + 1.178\,(1 + 0.0232\,i) - 0.0715\left(\frac{0.622 - 0.163\,i}{0.410^2} - \frac{0.378\,i - 0.163}{0.820}\right) - \frac{2 \times 0.0715}{0.410^2}\,(A_v + B_v\,i) = 0 \qquad (91a)$$

from which

$$0.1780 + 0.0273\,i - 0.2788 + 0.1023\,i - 0.8507\,A_v - 0.8507\,i\,B_v = 0 \qquad (91b)$$

Separating the real and imaginary terms leads to

$$\text{Real terms: } -0.1008 - 0.8507\,A_v = 0 \qquad (92a)$$

$$\text{Imaginary terms: } 0.1296 - 0.8507\,B_v = 0 \qquad (92b)$$

—from which $A_v = -\,0.1185$ and $B_v = 0.1524$. Thus, the complex term, $f_4 = -\,0.1185 + 0.1524\,i$, determines (for $k = 0.41$ according to Eq. 73) the lift force F_v acting on the windward chord.

In the second test, observed values included: Natural frequency (wind off), $\omega_2 = 1.740$; frequency of the excited torsional motion, $\omega = 1.670$; critical

velocity, $v_c = 127$ ft per sec; and logarithmic decrement of structural damping, $\delta_s = 0.009$. The corresponding value of k is $k = \frac{\omega\, b}{v_c} = \frac{1.67 \times 45}{127} = 0.591$. Proceeding in the same manner as before, it was found that $f_4 = -0.1006 + 0.1392\, i$.

Inspection of the values of f and f_4 shows that the parameters A_v and B_v diminish slightly with increasing k. For the purpose of the following investigation of model B it is satisfactory to assume that, within the range of k-values associated with the stability problems of models A and B, the parameters A_v and B_v vary proportionately with k.

Section Model Mounted on Free Springs (Model B).—The design characteristics for model B are the same as those for model A. Observed test values included: Natural vertical frequency, $\omega_1 = 0.814$; natural torsional frequency, $\omega_2 = 1.320$; and logarithmic decrement of structural damping, $\delta_s = 0.003$. The values of s, μ, and κ are the same as those for model A. For model B, however, $\frac{\omega^2{}_2}{\omega^2{}_1} = X = 2.62$ and $g = \frac{\delta_s}{\pi} = 0.001$.

The procedure in determining the natural frequency and the critical velocity at which excited motion begins is essentially the same as shown in Section 5, "Illustrative Problem," whereby the extended stability condition (Eq. 82) must be applied. A sequence of k-values is chosen, for each of which the expression Δ_o is computed by Eq. 63, assuming $D = 1$ for section model investigations. In adding to each Δ_o the additional term depending on f_4, the complete Eq. 82 is obtained, which finally yields the two equations from which the parameters X and Z corresponding to the assumed value of k can be calculated. On drawing the X-diagram and the Z-diagram as demonstrated in the previous section, the value of k_c and the corresponding value of Z can be found. The computation, using $k = 0.73$ is shown subsequently.

Determination of Δ_o from Eq. 63.—Introducing the numerical values for s, μ, κ, and g in Eq. 63 and using $f_1 = 0.5614 - 0.1233\, i$, $f_3 = 0.4386 - 0.1233\, i$, and $D = 1$, and separation of the real from the imaginary terms gives

$$0.763\, X\, Z^2 - 0.7560\, X\, Z - 0.8251\, Z + 0.8148 = 0 \quad \text{(93a)}$$

and

$$0.00153\, X\, Z^2 + 0.03121\, X\, Z + 0.02816\, Z - 0.06026 = 0 \quad \text{(93b)}$$

The next step is to compute the terms in Eq. 82 which depend on f_4. Thus,

$$\frac{2\,\mu}{k^2}(1 - Z) = \frac{0.109}{0.5329}(1 - Z) = 0.2045\,(1 - Z) \quad \text{(94a)}$$

and

$$\frac{\mu\,\kappa}{k}(-1 + X\,Z)\, i = \frac{0.763 \times 0.545}{0.73}(-1 + X\,Z)\, i$$
$$= 0.0570\, i\,(-1 + X\,Z) \quad \text{(94b)}$$

From the two values for f_4 computed previously the expression f_4 for $k = 0.73$ is derived—namely, $f_4 = -0.087 + 0.129\, i$. Multiplying Eqs. 94 by this

value of f_4 gives

$$-0.01779 + 0.01779\,Z + 0.02638\,i - 0.02638\,i\,Z \qquad (95a)$$

and

$$0.00496\,i - 0.00496\,i\,X\,Z - 0.00735\,X\,Z + 0.00735 \qquad (95b)$$

Adding the real terms of Eqs. 95 to Eq. 93*a*, and the imaginary terms to Eq. 93*b*, finally leads to

$$0.763\,X\,Z^2 - 0.7633\,X\,Z - 0.8073\,Z + 0.8044 = 0 \qquad (96a)$$

and

$$0.00153\,X\,Z^2 + 0.02625\,X\,Z - 0.00178\,Z - 0.02892 = 0 \qquad (96b)$$

From Eq. 96*b*,

$$X\,Z = \frac{0.00178\,Z + 0.02892}{0.00153\,Z + 0.02625} \qquad (97)$$

Substituting Eq. 97 in Eq. 96*a* yields the quadratic equation,

$$Z^2 - 1.3360\,X + 0.3464 = 0 \qquad (98)$$

having the roots $Z_1 = 0.943$ and $Z_2 = 0.392$. From Eq. 97 the associated values of X are computed—that is, $X_1 = 1.04$ and $X_2 = 2.68$.

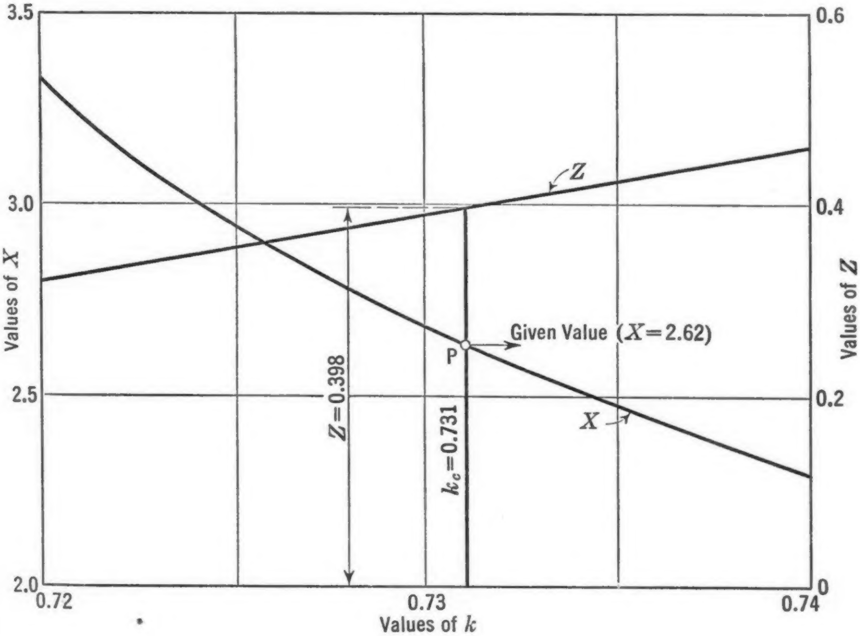


FIG. 14

In the same manner, the roots Z and X were computed for $k = 0.72$ and $k = 0.74$. The X-diagram and the Z-diagram built up from the X_2-values and the Z_2-values are shown in Fig. 14. The ratio $X = \frac{\omega^2{}_2}{\omega^2{}_1} = 2.62$ determines

point P on the X-diagram which finally defines the critical values $k_c = 0.731$ and $Z = 0.398$—from which the flutter frequency, $\omega = \frac{\omega_1}{\sqrt{Z}} = \frac{0.814}{\sqrt{0.398}} = 1.29$, and the critical wind velocity, $v_c = \frac{\omega\, b}{k} = \frac{1.29 \times 45}{0.731} = 79.5$ ft per sec, are obtained. Reduced to model scale, therefore, the computed and observed values compare as follows:

Value	Computed	Observed
ω	11.17 (107 cycles per min)	109 cycles per min
v_c	9.2 ft per sec	9.4 ft per sec

—showing that the theoretical results are in good agreement with the observed values of ω and v_c.

Observing that the frequency ω of the excitation lies very close to the natural frequency ω_2, it is concluded that the torsional component of the coupled vibration dominates the motion. The determination of the ratio $\frac{u_1}{u_2}$ yields $\frac{\eta}{\phi} = 0.134$—that is, the angular amplitude is about seven times as great as the vertical amplitude. Accordingly, the lateral shift of the center of rotation is small, its theoretical maximum value amounting to 50 in., or about 2/3 in. when reduced to model scale. The observed windward shift of the center of rotation was 3/4 in. The close accord between the theoretical and the measured values of the flutter characteristics indicates that the premises on which the theory is based—namely, the supposed mathematical form of F_v and the independence from F_v of the flat plate forces that act simultaneously on the structure—are approximately correct.

The illustrative example treated in this section offers the opportunity to demonstrate the marked effect of the configuration of the leading edge upon the dynamic stability of the bridge structure. It may be recalled that the lift force F_v associated with the vortex discharge at the windward chord always effects reduction of the critical velocity v_{c0} which would occur when F_v approaches zero (pure flutter). The ratio $\frac{v_c}{v_{c0}}$, therefore, offers an excellent measure for classifying the degree of the inherent aerodynamic stability of any cross section, and will be referred to as the "stability factor of the cross section." Actually $\frac{v_c}{v_{c0}}$ varies somewhat with the other dynamic properties, such as stiffness, mass, and the like, but these variations are of such minor degree that $\frac{v_c}{v_{c0}}$ may be considered a characteristic of the cross section.

As an illustration, calculations have been made to establish the stability factor of model B. Applying Eq. 59 to model B, the flutter speed is found to be $v_{c0} = 18.6$ ft per sec (model scale). Hence, the stability factor becomes $\frac{v_c}{v_{c0}} = \frac{9.2}{18.6} = 0.49$. Therefore, the particular shape of the chord and the

adjacent part of the deck structure of model B accounts for the reduction of v_c to one half of its possible optimum, v_{c0}. The behavior of the model of the Tacoma Narrows Bridge discussed in Section 5 and having the stability factor $\frac{v_c}{v_{c0}} \doteq 1$ proves that the efficiency of the cross section can be materially improved by an adequate design of certain structural details based on the information obtained from comparative tests in the wind tunnel.

The marked effect of the particular shape of the leading edge on the dynamic stability of a truss-type suspension bridge has already been disclosed in the tests made by F. B. Farquharson, M. ASCE, at the University of Washington Laboratory. The importance of an adequate shape of chord and adjoining deck structure has been repeatedly stressed in his reports on these tests. The theory developed hereinbefore provides a procedure of judging quantitatively the efficiency of any configuration of the leading edge.

7. Summary and Conclusion

The primary objective of this paper is to outline a rational method of approach for solving the problem of self-excited motion due to wind action of truss-stiffened suspension bridges and bridges with extremely shallow stiffening girders. The analysis developed in this paper is confined to the investigation of the effect of horizontal wind.

The method is based on the conception that these types of bridges behave aerodynamically like a flat plate acted on by air forces distributed over the width of the plate, and by additional air forces represented by a lift force F_v at the windward chord. The flat plate air forces are assumed to be identical with the aerodynamic air forces that originate on an oscillating streamlined thin flat plate under horizontal wind action, and for which the mathematical expressions are well established. The lift force F_v depending in magnitude and phase angle on the particular shape of the leading edge must be determined experimentally on a section model of the type of cross section under consideration.

The theory suggested herein explains all the characteristic phenomena which have been observed on section models and full-scale models of truss-type cross sections with closed deck acted on by a horizontal wind stream. Good quantitative agreement between theory and experiment is demonstrated in two cases. Other test results useful for checking the theory have not been available to the author.

In closing, the following remarks may be of value in properly judging the theory outlined and for its significance within the scope of the whole problem of self-excitation in suspension bridges.

The mathematical solution of the problem of dynamic instability of suspension bridges, as herein developed, is based on well-established principles of dynamics. The solution is sufficiently general so that the outlined method may serve as a pattern in solving all those problems of dynamic instability of suspension bridges which, mathematically speaking, reduce to a system of homogeneous linear equations. Assuming that the coefficients which deter-

mine magnitude, distribution, and timing of the air forces are known, the response of any type of suspension bridge structure (including also bridges with double lateral systems, tower stays, and other stiffening elements) can be determined theoretically and an exact correlation between the behavior of a section model and the prototype readily can be established.

The special theory discussed herein covers those types of suspension bridges which may be considered of importance for future suspension bridge design—namely, truss-stiffened bridges and bridges with extremely shallow stiffening girders. Girder cross sections of any considerable depth-to-width ratio exhibit such high vulnerability to aerodynamic forces that, in the author's opinion, they cannot compete successfully with these other types.

The introduction of the lift force F_v in the analysis of the problem is an attempt to explain the phenomena of self-excitation by the combined action of the known flat plate air forces and of a hypothetical dynamic force F_v without referring to the actual mechanism which generates that force. The concept of hypothetical forces has been used repeatedly in solving even basic problems in physics. Success or failure depends only on the proper choice and the adaptability of the mathematical expressions for such forces introduced in the analysis. Further tests may justify the suggested mathematical form or may indicate some necessary refinement. Close collaboration between theoretical and laboratory research may lead to a satisfying solution of the particular problem discussed, thus establishing a mathematical method for the solution of related problems of self-excitation in suspension bridges.

8. Acknowledgments

The theoretical studies reported herein are part of an extensive investigation of the problem of vibration in suspension bridges sponsored by the Advisory Board on Investigation of Suspension Bridges of the Public Roads Administration. They represent the essentials of two reports submitted by the author to the members of the Advisory Board, dated November 5, 1945, and February 27, 1947. The author is especially indebted to Professor Farquharson, director of the Engineering Experiment Station, University of Washington, and to George S. Vincent, M. ASCE, highway bridge engineer, Public Roads Administration, for their cooperation and constructive criticism of his studies.

APPENDIX. NOTATION

The following letter symbols, adopted for use in the paper, and in its discussion, conform essentially to American Standard Letter Symbols for Science and Technology (ASA—Z10) prepared by a Committee of the American Standards Association, with Society participation:

A = cross-sectional area; A_c = cross-sectional area of cables;

A_v = a coefficient in the expression (Eq. 71) for the lift force F_v;

$a_1, a_2, \cdots, a_n$ = coefficients in Φ-expansion for η;

B_v = coefficient in Eq. 71 for the lift force F_v;
b = half bridge width;
$C(k)$ = a complex function of k (defined by Eq. 5);
D = defined by Eq. 45;
d = depth of girder;
E = modulus of elasticity:
 E_c = modulus of elasticity of the cables;
 E_f = modulus of elasticity of the stiffening frames;
e = base of natural logarithms;
F = a periodic force:
 F_L = a resultant lift force acting on a vibrating flat plate;
 F_v = a periodic lift force acting on the leading edge;
$F(k)$ = a transcendent function of k (Table 1);
f = air force function:
 f_1, f_2, f_3 = air force functions defined by Eqs. 6;
 f_4 = air force function defined by Eq. 72;
$G(k)$ = a transcendent function of k (Table 1);
g = damping coefficient:
 g_1 = vertical vibrations;
 g_2 = torsional vibrations;
H = cable tension:
 H_i = cable tension due to inertia forces;
 H_w = cable tension due to dead weight w;
h = sag of cable in main span;
I = moment of inertia; I_f = moment of inertia of total stiffening frames;
i = imaginary unit $(= \sqrt{-1})$;
K = a parameter defined by Eq. 28;
k = a dimensionless ratio $\left(= \frac{\omega\, b}{v}\right)$; k_c = critical value of k;
L = length of total bridge:
 L_e = length defined by Eq. 13;
 L_m = length of the main span;
 L_s = length of the side span;
l = a distance;
M = a moment; M_L = resultant lift moment acting on a vibrating flat plate;
m = mass of bridge per unit length;
N = factors of normalization of the functions Φ;
n = an integer;
p = period of oscillation;
Q_j = the generalized force corresponding to the generalized coordinate q_j;
q_j = generalized coordinates:
 $\dot{q}_j$ = first derivative with respect to time;
 $\ddot{q}_j$ = second derivative with respect to time;
R_1 and R_2 = magnitudes in Eq. 38 depending on the air forces F_L and M_L;
r = mass radius of gyration;
s = a substitution factor $(= 2\,\pi\,\rho\, b)$;

T = kinetic energy;
t = time;
u_1, u_2 = dimensionless parameters determining the contributions of the vertical and torsional vibrations to the resultant motion;
V = potential energy;
v = wind velocity; v_c = critical wind velocity;
W = work done by the external forces;
w = dead weight;
X = a substitution factor $\left(= \frac{\omega^2{}_2}{\omega^2{}_1}\right)$;
x = a rectangular coordinate;
Z = a substitution factor $\left(= \frac{\omega^2{}_1}{\omega^2}\right)$;
α = angle of attack;
Δ = a determinant;
δ = logarithmic decrement of damping:
  δ_a = logarithmic decrement of aerial damping;
  δ_s = logarithmic decrement of structural damping;
η = dimensionless vertical amplitude $\left(= \frac{\bar{\eta}}{b}\right)$;
  $\dot{\eta}$, $\ddot{\eta}$ = first and second derivatives with respect to time;
  $\bar{\eta}$ = vertical or bending amplitude;
  $\dot{\bar{\eta}}$, $\ddot{\bar{\eta}}$ = first and second derivatives with respect to time;
θ = difference between phase angles of the vertical and torsional components of the motion;
κ = a substitution factor $\left(= \frac{2\,r^2}{b^2}\right)$;
μ = a substitution factor $\left(= \frac{s\,b}{m}\right)$;
ρ = mass density of air (assumed 0.00238 slug);
Φ = a function of x;
ϕ = torsional amplitude; $\dot{\phi}$, $\ddot{\phi}$ = first and second derivatives with respect to time;
ψ = a time function; and
ω = flutter frequency:
  ω_1 = frequency of the vertical component of motion;
  ω_2 = frequency of the torsional component of motion.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852

REPORTS

ADVANCES IN SEWAGE TREATMENT AND PRESENT STATUS OF THE ART

FOURTH PROGRESS REPORT OF THE COMMITTEE OF THE SANITARY ENGINEERING DIVISION ON SEWERAGE AND SEWAGE TREATMENT

This report of the Committee on Sewerage and Sewage Treatment covers the years 1946 and 1947—a period of readjustment and inflation of prices. In the field of sewerage and sewage treatment, a shortage of material, labor, and experienced engineers and draftsmen is still prevalent. There is still a large backlog of delayed civilian work, such as sewers, sewage treatment works, and garbage and waste disposal works, which has been shelved because of high construction costs. Sizable contracts proceed slowly, because of lack of materials. However, the wise community will continue to study its problem and develop plans for future use, in order of urgency. With the high prices for contract work (often more than twice pre-World War II prices), available bonding power does not cover as much actual work. Consequently the necessity of the work must be thoroughly scrutinized.

Sewage Research

The annual review of the literature on sewage and waste treatment and stream pollution by the Committee on Research of the Federation of Sewage Works Associations (1)[1] continues to summarize the art each year from the research, laboratory, and operating standpoints. The National Council for Stream Improvement, Incorporated (2), is studying problems of disposal of wastes from de-inking waste paper, strawboard and other board manufacture, and other types of pulp and paper processes. Under the Public Health Service Act of 1944, the National Institute of Health (3) approved in 1946–1947 twenty-four federal grants for research in the sanitation field, calling for an

Note.—Please forward all comments on this report directly to Chairman Langdon Pearse, 910 South Michigan Ave., 7th Floor, Chicago 5, Ill. Progress reports are published in *Proceedings* only.

[1] Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography (see Appendix).

expenditure of $188,854. The aid extended varies from $892 to $33,830 per project. Included is an item of $10,832 for the study of the efficacy of small septic tanks. Apparently there is need for thorough scrutiny of projects.

United States Public Health Service

In a country-wide survey of 78,000,000 people in 5,800 communities, the United States Public Health Service (4) finds need for a sanitation program estimated to cost $7,800,000,000. Of this, $2,055,000,000 is for expenditures on water and sewage by New York, N. Y., Philadelphia, Pa., Chicago, Ill., Detroit, Mich., St. Louis, Mo., Pittsburgh and Allegheny County in Pennsylvania and Denver, Colo. Distribution of these requirements by population groups and type of project are as follows:

Description	Cities more than 100,000	Communities 200 to 100,000	Total
Water works	$1,174,607	$1,094,375	$2,268,982
Sewerage	1,912,000	1,836,604	3,748,853
Garbage and refuse	61,455	105,071	166,528
Rural sanitation	......	......	1,650,220
Total			$7,834,581

(Costs are in thousands of dollars.) Rural needs include water supply repairs or betterments for 6,000,000 homes ($800,000,000), new privies or repairs ($400,000,000), and about 1,700,000 homes with indoor plumbing need new outdoor disposal systems or repairs to old ($400,000,000).

During the peak of the activities of the United Nations Relief and Rehabilitation Administration in controlling environmental sanitation in various countries in Eastern Europe, Asia, and Ethiopia (5) between November, 1944, and July, 1946, sixty-five sanitary engineers were on duty, of whom fifty two were from the U. S. Public Health Service.

During 1946, the U. S. Public Health Service cooperated with a committee of citizens, the State of Illinois, and various municipalities and authorities in the Chicago-Cook County Health Survey. This included all the municipal activities relating to public health, including water, sewage, milk, food, hospitalization, and laboratories.

Federal Legislation on Pollution

All the bills for pollution control presented to the Seventy-ninth Congress, Second Session, and to the Eightieth Congress, First Session, failed to pass. The legal and historical background of stream pollution control was discussed by the Committee on Sewage Disposal, Public Health Engineering Section, American Public Health Association, in two reports (6)(7) covering the subject up to October, 1947. One bill, known as the Water Pollution Control Act (8), prepared by Philip B. Fleming, M. ASCE, was passed by the Senate on July 16, 1947, and can still be considered by the House of Representatives in the Second Session. This bill was based on the Barkley-Taft (9), Spence, and Elston bills, which were identical. As rewritten, the bill places the

pollution control under the Surgeon General of the U. S. Public Health Service and the construction aspects under the Federal Works Agency, and authorizes a maximum annual federal work appropriation for loans-in-aid amounting to $100,000,000. An editorial in the *Engineering News-Record* (10) analyzed the bill and recommended its defeat. W. J. Scott, M. ASCE (11), and M. LeBosquet, Jr. (12), have reviewed the bills before Congress in 1947. Mr. Scott does not believe that federal financial aid in the form of grants-in-aid or loans is a "must." Mr. LeBosquet outlines developments leading up to and including the bills before Congress.

However, one bill granting powers for the control of pollution was passed by Congress in 1946 (13), authorizing the Secretary of the Interior to take steps for the conservation of wild life and its rehabilitation on streams or other bodies of water impounded, diverted, or controlled by any department or agency of the United States or by any public or private agency under federal permit, provided that any rules and regulations shall not be inconsistent with the laws for the protection of fish and game of the states in which such area is situated. Authority is granted for a wide investigation and report to Congress. The Tennessee Valley Authority is exempted from the provisions of the act.

Other Federal Legislation.—The Eightieth Congress, First Session (14), approved an interstate compact between Connecticut, Maine, Massachusetts, New Hampshire, and Vermont relating to the control and reduction of pollution of the streams and waters therein. Prior thereto, Connecticut, Massachusetts, and Rhode Island had ratified such a compact.

Progress on State Pollution Control

Legislation and Action.—A. Anable and R. P. Kite (15) note that thirty-seven states, six Canadian provinces, Hawaii, and Alaska have laws regulating the discharge of industrial wastes into a stream. The legislative activity to accomplish this result has been divided among the various decades as follows:

Period	No. of states
Prior to 1910	4
1911 to 1920	8
1921 to 1930	15
1931 to 1940	9
1941 to 1947	22

Presentation and approval of plans for waste treatment are required by thirty-three states and two territories. Practically all state laws are enforceable and provide penalties for noncompliance with antipollution requirements; only two states have laws that do not. Forty-two states make no exceptions. Five states make exemptions for mines and concentrators; one, for war industries.

In the New England States, stream pollution abatement (16)(17)(18)(19)(20)(21) is proceeding. In New York State, a report of a state committee (22)(23) resulted in an act authorizing state aid for planning. In Pennsylvania (24), West Virginia (25), Virginia, and Indiana (26), extensive programs are under way. Washington (27), Oregon (28), and California (29) are also active. M. M. Cohn, M. ASCE (30), reviews the status in thirty-six states.

Pollution Surveys

A few pollution surveys are of interest, such as those of the Allegheny County, the Interstate Commission on the Delaware River Basin, and the Interstate Commission on the Potomac River Basin.

Allegheny County.—In Allegheny County the Sanitary Authority (31) now comprises ninety nine of the county's one hundred and twenty-eight communities and thirty industries, including more than 85% of the population of the area. The daily discharge of sewage from thirty-three main outlets exceeds 200 mgd (32). Pollution abatement is to be considered (31) on a metropolitan level, rather than for individual communities. Under its enabling act, the authority can prepare a report, but cannot draft plans and specifications without the consent of the municipalities.

Interstate Commission on the Delaware River Basin.—"Incodel" (33) describes a decade of planned progress in water pollution control on the Delaware River Basin, wherein forty-seven communities in Pennsylvania and New Jersey have spent more than $10,000,000 installing sewage collection and treatment facilities. Philadelphia, Pa., Camden, N. J., and Port Jervis, N. Y., are actively working on plans. In the water supply field, following the principle of "equitable division," New York, New Jersey, and Pennsylvania have entered into statutory interstate agreement. Among other interests are water conservation (a war measure), the Schuylkill River restoration, flood control, and navigation.

Interstate Commission on the Potomac River Basin.—The Fifth Annual Report of the Interstate Commission on the Potomac River Basin (34) discusses the problem of controlling and abating pollution and the need of constructing an impounding dam on the Savage River to increase the dry weather flow by about 100 cu ft per sec. The commission has cooperated in arranging for the treatment of the sewage of a part of the Washington Suburban Sanitary District area by the District of Columbia. The commission has also participated in securing the passage of a pollution control act in Virginia (March 30, 1946).

Notable Projects

Among the larger projects placed under construction in the postwar period are those at Boston, Mass., and Los Angeles, Calif.

Boston, Mass.—K. R. Kennison, M. ASCE, reports (35) that Boston Harbor receives at present (1947) about 250 mgd of raw sewage, from three main outlets: (*a*) Moon Island, the outlet of the old main drainage system of the City of Boston; (*b*) Deer Island, the outlet of the north metropolitan system; and (*c*) Nut Island, the outlet of the south metropolitan system. The Moon Island outlet is located about in the center of the inner harbor and within 1 to 3 miles of bathing beaches. Raw sewage is discharged on the outgoing tide. At the other two outlets, the discharge is continuous. Treatment works are planned for each outlet. Construction is under way on the Nut Island plant, which is designed to remove floating solids and grease and to reduce the suspended solids to the extent possible with 1½-hour settling, preceded by 20-min aeration; chlorination of effluent during the recreational season; digestion

of sludge and utilization of gas; and discharge of digested sludge through the outfall to tidewater.

Los Angeles, Calif.—To clean up 12 miles of bathing beaches, Los Angeles (36) is constructing a sewage treatment plant and outfall. Temporarily the sewage is being chlorinated, using seven 6,000-lb-capacity chlorinators. The permanent plant comprises bar screens (1-in. opening), comminutors, grit chambers, primary settling tanks, aeration tanks (4.31-hour period on 245 mgd average dry weather flow), and final settling tanks (2.5-hour period). The sludge will be digested with heat control, elutriated, conditioned with ferric chloride, dewatered on vacuum filters, and heat-dried by flash driers for fertilizer, or burned. Chlorination equipment is planned for both prechlorination and postchlorination.

Reports on Projects

A number of reports on projects, proposed or about to commence, have reached the Committee, which are noteworthy. These concern Columbus, Ohio; the East Bay Cities in California; Merrimack River Valley (New England); Sanitation Districts Nos. 3, 15, 16, and 17 of Los Angeles County, Calif.; Portland, Ore.; and Orange and Santa Clara counties in California.

Columbus.—In July, 1946, a report (37) was made on sewage treatment at Columbus, describing present conditions and the required rehabilitation of the existing activated sludge plant. Because of the low dry weather flow of the Scioto River, the highest practicable degree of sewage treatment is needed. The continued use of the activated sludge process is recommended, with effluent chlorination when necessary. The increasing volume of sewage flow due to industrial and air conditioning cooling water from wells suggests the separation of such water for direct discharge to the river. Storage of river water for let down at time of low flow is suggested.

East Bay Cities.—The East Bay Cities on the easterly shore of San Francisco Bay (38)(39) plan to separate the sanitary sewage from the storm water in most of the existing combined areas, reducing the maximum flow to about 100 mgd. The area sewered is about 55.6 sq miles, with a 1940 population of 447,480. Treatment proposed in San Francisco Bay, where deep waters and a strong current exist, consists of sedimentation (1 hour as of 1970) with chlorination. Flocculation for 20 min may be added if necessary. In other waters, complete treatment is indicated. The sludge will be digested with heat control, elutriated, and partially dewatered with vacuum filters.

Merrimack River Valley.—Under Chapter 62, Resolves of 1945, of the Commonwealth of Massachusetts, a joint board has investigated the disposal of sewage in the Merrimack River Valley in Massachusetts. T. R. Camp, M. ASCE (40), describes in detail the pollution by Lowell and Lawrence and surrounding territory, of which 77.5% of the biochemical oxygen demand (B.O.D.) is due to industrial wastes. Regional treatment in four main projects is recommended, with activated sludge treatment at Lowell and Lawrence, and sedimentation at Haverhill and Newburyport Harbor, all effluents to be chlorinated. The cost of construction is estimated at $27,581,100, with an annual cost of $1,339,020, of which 58% will be assessed to industry. Of the total

cost, 61.2% is attributable to volume; 15.4%, to B.O.D.; and 23.4%, to suspended solids.

The river has a total drainage area of 5,006 sq miles, of which 3,798 sq miles are in New Hampshire. At Lowell, flows of less than 700 cu ft per sec may occur 1% of the time; and flows of less than 1,500 cu ft per sec, 10% of the time. The entire district has an equivalent population of 1,271,985 (1947), of which the state census in 1945 showed 343,316 resident population (estimated in 1947 at 374,010). The sewage flows, in gallons per capita per day, were estimated, respectively, as: Lowell, 60; Lawrence and Methuen, 70; Haverhill, 90; Amesbury, 65; Newburyport, 70; and Andover, 102. An unusual feature is an analysis of the effects of escape of sanitary sewage through storm water overflows. With an interceptor designed for twice the dry weather flow, 50% of the sewage will overflow during a storm of 0.03 in. per hr; 82%, at 0.10 in. per hr; and 96%, at 0.50 in. per hr. The report advocates interceptors for the peak dry weather flow, with a small margin of safety against overflows in dry weather. Separation of sanitary sewers from storm drains is recommended.

In the Lowell and Lawrence works, the pH runs from 8.5 to 10.5. This will be adjusted to pH = 8 by the use of CO_2. Flocculation tanks and grease separators are provided in all the sewage treatment works. All primary settling tanks will be long and narrow (length at least ten times width) with 5-ft water depth, providing detention periods of 1.1 hours for average flow. An aeration period of 6 hours is proposed, except for 8 hours at Lawrence. Heated sludge digestion tanks, with elutriation and vacuum filters, are proposed for the larger plants. Because of various local difficulties and objections, barging liquid sludge to sea was not recommended. Hauling sludge cake and dumping are believed more economical than incineration.

Pasadena, South Pasadena, Alhambra, and San Marino, Calif.—A M Rawn, M. ASCE, reports (41) that following the arrangement in 1942 for disposal of sludge in the trunk sewers of the County Sanitation Districts of Los Angeles County, the cities of Pasadena, South Pasadena, Alhambra, and San Marino have recently abandoned their joint plant and discharge all their sewage into the county system, at an estimated saving over 40 years of around $4,000,000. He further outlines (42) the growth of the Los Angeles County works and the plans for future intercepting sewers and sewage treatment in Sanitation Districts Nos. 3, 15, 16, and 17 of Los Angeles County, comprising the central and western sections of the San Gabriel Valley, including eight incorporated cities and an equal number of densely settled unincorporated towns.

Portland.—Portland (43), located on both banks of the Willamette River just above its confluence with the Columbia River, is sewered on the combined plan. There are sixty-two separate outlets—fifty one into the Willamette River (low flow, 4,000 cu ft per sec), and eleven into a slough tributary to the Columbia River (summer flow, 125,000 cu ft per sec to 150,000 cu ft per sec; and low flow in fall and early winter, 75,000 cu ft per sec). The sewage flow is to be collected at a single point for treatment (removal of floating solids and coarse suspended matter) and discharge to the Columbia River. Treatment comprises coarse racks with 1-in. openings, grit removal, sedimentation (1.5 hours at dry weather flow and 0.6 hour at maximum flow), and two-stage

heat-controlled sludge digestion (2.5 cu ft per capita). The fully digested sludge is to be discharged at a low rate into the outfall sewer. The added organic load from digested sludge and the effect on the dissolved oxygen in the Columbia River are very small.

Orange County.—The sewage disposal problems of Orange County (44) arise from an area of 800 sq miles, including thirteen cities with a population of 114,000. The total population is about 200,000. The county has an ocean shore line of about 42 miles. Domestic sewage is estimated at 70 gal per capita per day, except at Newport Beach (140 gal per capita per day). Industrial wastes (21.5 gal per capita per day) are chiefly from beet sugar manufacture and the processing of vegetables and fruit. Ocean currents were studied in detail to locate an ocean outfall. Primary sewage treatment with grease removal and sludge digestion is proposed for a main outfall (15,000 ft from shore) and a minor outfall (2,000 ft from shore). For an inland plant, settling plus oxidizing ponds with recirculation is proposed. The cost of reclaiming sewage for a water supply was found excessive.

Santa Clara County.—The report on the sewage disposal problems of Santa Clara County (45) concerns an area of 1,305 sq miles, with a 1940 population of 175,000, of which 109,000 reside in incorporated municipalities. The county was divided into seven sanitation districts, tributary to a proposed central sewage treatment works. There are some sixty-five industrial establishments, mostly processing fruit and vegetables. The total equivalent population (including domestic) for 9 months averaged about 420,000, with a peak 3-month average of 750,000. The area drains into Lower San Francisco Bay. Complete treatment is recommended, comprising pre-areation, primary sedimentation, oxidation ponds, recirculation, sludge digestion tanks, sludge disposal lagoons, and chlorination.

Sewage Treatment Works at Military Installations

The Sub-committee on Sewage Treatment in Military Installations of the Committee on Sanitary Engineering, National Research Council (46), contacted one hundred and thirty-five sewage treatment works out of three hundred and ninety nine at military installations, and analyzed one hundred and thirty. Twenty-five navy sewage treatment works were included. The report is an excellent manual on such military works, covering the characteristics of the sewage (flow, B.O.D., suspended solids, grease, and pH); preliminary treatment; sedimentation; trickling filters (deep or shallow, with recirculation, or single or multistage); contact aeration; activated sludge; sludge digestion and disposal; and oxidation ponds. The sub-committee concluded that contact aerators are less desirable for use in military camps than trickling filters, that with present designs contact aerators were not superior to activated sludge treatment, and that activated sludge plants are not sufficiently flexible to handle the variations in flow and load occurring at military posts.

Grease Recovery

The grease content of sewage at military sewage treatment installations ranged (46*a*) from 33 ppm to 255 ppm, with occasional grab samples as high

as 1,000 ppm. With the installation of efficient grease traps at the source, combined with proper maintenance, the Salvage Branch of the Quartermaster Corps collected approximately 6,000,000 lb of grease per month.

At the New York City sewage works grease has been salvaged since June, 1943 (47). During 3½ years, the sale of 4,593,840 lb of skimmings produced $36,750.72. The wet material weighed about 60 lb per cu ft, the dry solids containing about 37% of ether soluble material.

The quantity of wet skimmings during 1946 for six plants varied from 11 lb per million gal to 25 lb per million gal, except at Wards Island, which had a low value of 5.7 lb per million gal. During 1947 the grease was sold under a modified contract whereby the city received 6% of the published price of "house grease" (48), resulting in prices varying from 0.6¢ per lb to 1.3¢ per lb. The contractor rejects skimmings contaminated with large amounts of sewage solids or mineral oil.

The only material containing grease now sold by The Sanitary District of Chicago is collected at the Racine Avenue Pumping Station, through which a large part of the wastes of Packingtown passes. The scum is removed from an exposed area in the intake chamber, of limited efficiency. The material is sold under contract, after advertisement. Only one bid has ever been received at a letting. The price has declined from 1.1¢ per lb in 1943–1944 to 0.25¢ per lb in December, 1947. The yearly collection has declined from 2,371,578 lb in 1944 to 373,829 lb in 1947. The material from the Racine Avenue station yields, on a dry basis, about 35% grease, containing 6% unsaponifiable matter.

The grease removal units (49) in the South Saint Paul, Minn., treatment works were designed to receive a large proportion of packing house wastes. They are seldom used, as aeration at the influent end made the grit chambers effective grease removal units. During the three years 1945 through 1947 the annual income from the sale of grease averaged $14,467. In the winter of 1941–1942, as much as 60 bbl of 30% grease were skimmed from the grit chambers daily. Thereupon, the packers increased their grease removal units and catch basins, thus reducing the amount collected at the sewage works.

Grease Recovery in the Meat Packing Industry.—E. N. Mortensen (50*a*) discusses the economic side of recovery of by-products and the proportion of income derived from by-products in the meat industry. He states that current design keeps the flowing-through velocity between 1 ft per min and 2 ft per min, and that mechanical skimming devices are justified in large operations.

Recovery of By-Products from Sewage Sludge

At the Bradford, England (51), sewage treatment works, the bulk of the grease is recovered from the deposited sludge, which is screened through ½-in. openings, pumped into vats heated by live steam to a temperature of from 180° F to 200° F, and acidified to be slightly alkaline by methyl orange. Cooking lasts about 40 min. The sludge is then forced into heated plate presses, with an initial pressure of 8 lb per sq in., the pressure being built up to 40 lb per sq in. in 24 hours and to 55 lb per sq in. in 48 hours. The presses operate on 3-hour sludging, alternating with 1-hour steaming. The cake contains about 16% grease on a dry solids basis. The effluent from the presses passes

to tanks in which the grease is separated by flotation. The press cake contains 14.6% moisture, 39.54% organic matter, 0.25% phosphoric acid, 2.13% organic nitrogen, and a trace of potash.

The crude grease is treated with 5% acid and cooked for 3 hours. When recovered by settling, the clarified grease contains about 70% wool grease and 30% domestic grease. Of the dry grease, 66% is saponifiable. Research developed a process for deodorizing and also for making various derived products.

Waste Oil

Collection in Cities.—At Baltimore, Md. (52), a contractor has collected from October 1, 1927, to December 1, 1943, 4,287,114 gal of material, or 263,860 gal per yr, at an average annual cost of $1,970.08 per yr. From October 16, 1944, to October 16, 1947, the city paid $2,150 per yr. Under a new 3-year contract, beginning in November 1, 1947, the city will pay only $775.00 per yr. The amount collected in 1944–1946 averaged 292,079 gal per yr.

In The Sanitary District of Chicago, at least nine firms collect waste oil from garages, filling stations, and factories, and pay therefor from 1¢ per gal to 2¢ per gal. The material is sold for road oil, fuel oil, reprocessing, or refining for reuse. One collector burns it as fuel in manufacturing lubricating oil. The amount handled is estimated to be about 1,800,000 gal per yr. Reclamation may include settling, steam evaporation to drive off gasoline, dehydration in vacuum, and filtering.

Collection by Railroads.—A number of railroads in the United States have individual services for collecting the used packing from journal boxes on freight and passenger cars. The material is packed in 55-gal drums with locked covers and shipped to a convenient central point for processing, which is frequently done by contract. The used cotton waste is placed in a washing machine and washed with hot, dirty oil and drained. This step removes about 50% of the oil in the waste. The waste is then washed with hot, clean oil and drained. After the second washing, the waste is centrifuged, dried with heat, mixed with new waste, impregnated with oil in vacuum tanks, and packed in clean drums for shipment. On one large railroad, approximately 400,000 lb of waste are processed monthly.

The oil drained from the cotton waste is pumped into steam jacketed settling tanks, with closed tops and inspection holes. The tanks are heated and then allowed to settle, usually overnight. The clean oil is drawn off and any water is discharged into the nearest watercourse or sewer. The bottom residue is discharged into a grease tank for renovation. As much as 50,000 gal of oil may be processed in a month on a large railroad. In such a batch operation, some oil may occasionally escape.

Oil Separation by Industry.—L. F. Oeming (53) notes oil waste recovery by separators of various types at Willow Run and at Flint, Mich., in the automobile industry, with notable river improvements. In The Sanitary District of Chicago, during World War II, oil wastes from the testing chambers in a large aeroplane engine factory were collected in a special sewer and discharged through an API (American Petroleum Institute)-type separator.

Maintenance of Equipment

In 1946 and 1947, the maintenance of equipment in sewage treatment works continued to be difficult and also expensive. Many essential materials are still scarce. In a series of articles, Mr. Cohn covers (54) the practice in operation of sewage works, and the maintenance of a wide range of devices. S. G. Hess, M. ASCE (55), lists suitable tools and equipment for sewage treatment works.

Gas Engines.—L. S. Kraus (56) outlines a maintenance program for 300-hp and 535-hp gas engines running on sewage sludge gas, operating 95% of the time, and generating 97% of the power requirements of the activated sludge plant at Peoria, Ill. Each engine is overhauled at a 6-month interval. A satisfactory lubricating oil should be used, with suitable piston rings, liners, and spark plugs. Bearing clearances and shims are inspected every 6 months.

Diffuser Plates

In its earlier reports (57)(58)(59), the Committee has discussed the behavior of diffuser plates and the maintenance problems involved. The investigations at Chicago (Southwest Works) have continued, both on diffusers and on air cleaners. In the United States some operators still prefer a diffuser of low permeability (rating under 25 cu ft per sq ft per min) and a low air rate (not exceeding 1 cu ft per sq ft per min). Others are considering plates having a rating of 80 to 120 cu ft per sq ft per min. Unless the air is properly cleaned and a suitable rate of air passage maintained, trouble apparently may develop with any type of porous diffuser. A special committee of the Federation of Sewage Works Associations is working on a manual on the subject.

Milwaukee, Wis.—J. L. Ferebee, M. ASCE, does not favor high rating plates to pass excessive volumes of air per plate per minute. The Milwaukee plant uses about 1.5 cu ft of air per gallon of sewage. The original installation (1925) had a ridge and furrow bottom. In the extension (1936), spiral flow aeration was installed with two rows of plates per channel. During the highly humid summer periods, air passage seldom exceeds 1 cu ft per plate per min. In 1945, the plate area was increased $33\frac{1}{3}$%. Even this proved insufficient to pass the air required at such critical periods. In a proposed extension, the use of four rows of plates per channel is being considered. In one half of the original Milwaukee (1926) plant, the plates initially installed (rated at 12 cu ft per plate per min, dry) are still giving service after 21 years. However, although the same type of plate is used in the replacements, they have a higher rating (upward of 25 cu ft per plate per min).

Chicago Sanitary District.—In the works of The Sanitary District of Chicago, the behavior of diffusers at the three activated sludge plants has varied. At the Southwest Works, the volume of air passed per plate averages about 4 cu ft per min. In 1946–1947, water mold growths on the surface of the diffuser plates were not troublesome, although they were visible in the influent half of one tank in late August, 1946. New 80-permeability plates were installed in both rows of all the unit tanks of Battery B during August and September, 1947. These were set in concrete containers and cemented in place. High relative humidity of the air has not been a serious factor with new plates, but it is

troublesome when the plates are badly clogged. In 1946, the pressure in the air main at the north wall of the Pump and Blower House averaged 7.5 lb per sq in.; the maximum daily average pressure was 8.0 lb per sq in. Through September, 1947, the average pressure was 7.6 lb, with a maximum of 8.2 lb. The maximum allowable pressure at the blower is 8.5 lb per sq in.

Since construction work started on Battery C, the dust content of the compressed air rose to about 3 mg per 1,000 cu ft of free air, from a prior average of from around 2 mg to 2.5 mg. After continuous service since 1939, the original air filters of the oil dipped type were replaced in 1947 by four new filter units of the electrostatic type. The dust content of the filtered air is now about 0.68 mg per 1,000 cu ft at an air velocity of 500 ft per min.

In the eight unit tanks of Battery A, various tests are still in progress. In June, 1945, tank 1 was equipped with 80-permeability plates, using four rows in the first bay and one in the remaining bays. These were replaced throughout in the summer of 1947 with new 80-permeability plates in two rows. In tank 2, with 40-permeability plates set in cast-iron containers with removable top frames, new plates were installed in the effluent half in April, 1946, to replace those installed in May, 1945. In tank 3, swing diffusers with tubes of various permeabilities, installed in December, 1945, are under test. In tank 4, 80-permeability plates, installed in May, 1945, were replaced in September, 1947, with new 80-permeability plates in two rows in all bays. In tank 5, in August, 1945, a complete set of slotted brass tubes (1,458 in all) were installed, equivalent approximately to one row of diffusers. In tank 6, four rows of 80-permeability plates were installed throughout in August, 1945. In tank 7, in May, 1945, 40-permeability plates were installed, in vertical plate holders. These were replaced in March–May, 1946, with 80-permeability plates, which still serve even though the pressure loss is high. In tank 8, during May, 1945, 80-permeability plates were installed in one row throughout. These were replaced with 120-permeability plates in August–September, 1947.

At the Calumet Works, the annual average iron content of the sewage increased from 6.3 ppm in 1946 to 8.2 ppm in 1947. However, the iron in the return activated sludge was almost the same (1946, 8.87 ppm, and 1947, 8.47 ppm). The annual average from 1938 to 1947, inclusive, is 9.52 ppm, with a maximum month of 14.51 ppm and a minimum month of 7.35 ppm. During 16 months of 1936–1937, the monthly average ranged from 12.60 ppm to 18.56 ppm. During the summer of 1946, new 80-permeability plates were installed in one row in all tanks except one. Since then, little cleaning of plates has been necessary, except on the plates at the influent end of the tanks. The pressure in the air main has averaged from 7.7 lb per sq in. to 7.8 lb per sq in. during the last 2 years.

At the North Side Works, the original plates are still in service after nearly 20 years, except in three tanks, where new plates of other makes were installed in one row per channel in 1933–1934. Each of the original plates is estimated to have passed approximately 10,000,000 cu ft of air. In 1947, 40-permeability plates were placed in one row of one tank, to replace plates installed in 1934. The air main pressure is now about 7.7 lb per sq in. in the operating gallery—an increase of approximately 0.4 lb per sq in. since 1932.

Cleveland, Ohio.—J. W. Ellms, M. ASCE, reports that the usual amount of cleaning of diffuser plates at the Easterly plant has continued. A large proportion of the plates originally installed are still in use. New plates are bought to replace plates broken in handling.

In 1946, at the Cleveland Easterly plant (60), the aluminum aeration manifolds were replaced. Stainless steel is being tried, together with perforated 2-in. pipe. At the Westerly plant, corroded aluminum plateholders in the grease aeration unit were discarded prior to 1946. At the Southerly plant, in the aeration unit handling 48.1% of the sewage flow, the diffuser plates and holders were removed and replaced by drilled pipe headers in the channels. G. E. Flower (61) describes the difficulties with air diffusion, since operation began in 1938. In 1939, the sewage contained about 100 ppm of iron. Carbon teardrop tubes (grade 20, permeability rating 26) clogged in 3 months. After cleaning with acid, 700 tubes were replaced. Clogging occurred shortly after, so the tubes are now cleaned twice a year. The hoisting cables also failed and other types of plates required removal and cleaning. In 1945, the aluminum plateholders were corroded so badly that perforated pipes were installed.

Ann Arbor and Pontiac, Mich.—At Ann Arbor (62), aluminum plateholders have disintegrated. The diffuser plate installation has been replaced by galvanized pipe manifolds, with cellulose capillary tubes for diffusers. W. W. Dubay (63) reports cleaning diffuser plates at Pontiac with carbon tetrachloride, using 3 gal per million gallons of sewage flow. This was fed into the suction side of the air compressor at the rate of 1 gal per 10 min.

Rockville Centre, N. Y.—The efficiency of the aeration tanks at Rockville Centre, N. Y., was increased in 1945 by lowering the air diffusers from a submergence of about 4 ft to a position just above the bottom.

Control of Sludge Index

Mr. Kraus (64)(65) controls the sludge index by adding digested sludge and digester overflow to the activated sludge in an aeration tank and aerating until the whole is converted into activated sludge. Such low index sludge is then added to a high index sludge to obtain the desired mixed liquor. This interchange process may be made at a high rate over a 6-hour period each day, and at a low rate for the balance of the day. By removing large quantities of digester overflow and mixing them into the aeration system, the problem of handling supernatant liquor is eliminated. The load capacity of the activated sludge plant is increased, as the procedure is thought to eliminate the bulking problem.

Sludge Disposal

Although cities in England were shipping liquid sludge to sea in sludge boats prior to 1900, Cleveland appears to have been first in the United States to discharge digested sludge through the effluent outfall. According to Mr. Ellms from the start of the Westerly plant in 1923 (then serving about 90,000 population) the digested sludge from the Imhoff tanks was discharged through the plant outfall into the lake about 1,000 ft from shore, until 1938, when vacuum

filtration and incineration of the sludge were begun. The ash is now discharged into the outfall.

A new trend in sludge disposal has developed at Boston and at Portland. At Boston, Mr. Kennison proposes to discharge digested sludge on the outgoing tide into the fairway between Deer Island and Long Island. Portland proposes to discharge digested sludge from treatment works into the Columbia River.

Effect of Digestion on Sewage Sludge

Mr. Rawn and E. J. Candel (66) experimented with four stages of sewage sludge digestion and found that it effects a marked reduction in the B.O.D. and content of coliform group bacilli, as well as a radical change in sludge characteristics. Such digested sludge can be added to primary effluent, prior to disposal by dilution.

B.O.D. tests indicated a reduction from 11,100 ppm to 955 ppm, or 91.5%, in 60 days. The return of one volume of settleable solids to 350 volumes of effluent raised the average settleable solids of the primary effluent from 6.4 ml per liter to 7.9 ml per liter, or 23%. The average coliform group density of the raw sludge was 1,504,000 per ml. The primary effluent averaged 51,000 per ml, and the digested sludge contained 1,100 per ml. The chlorine demand of raw sludge varied from 950 ppm to 1,000 ppm. After 60-day digestion, the demand for sludge was 600 ppm, and for primary effluent, 36 ppm.

Messrs. Rawn and Candel conclude that, in the absence of sludge banks:

> "* * * there appears to be no valid reason why 11-day-old or 60-day-old, digested sewage sludge may not be disposed of by dilution along with the primary clarifier effluent if conditions permit disposal of the latter with safety."

Sludge Lagoons

The Committee on Sewage Disposal of the Engineering Section of the American Public Health Association (7) has reviewed the use of sludge lagoons both by municipalities and industry. Apparently in the United States there are about eighty-two sludge lagoons (67). Recently C. G. Hyde and G. L. Sullivan, Members, ASCE (45), have recommended the use of lagoons for sludge disposal on the Santa Clara County project. Although frequently used as an emergency outlet for excess sludge, lagoons have been used continuously at certain plants for more than 20 years. Lagoons may receive raw settled solids or digested settled solids. In some cases, secondary sludge from trickling filters, or excess activated sludge, is added to the primary sludge.

When receiving raw solids, a lagoon may be difficult to control in a northern climate and may give rise to unpleasant odors, particularly in the late spring, when the sludge temperature begins to rise above 60° F and the sludge held through the winter starts to decompose. When receiving digested solids, a lagoon becomes a storage basin and can be loaded up over a period of years and then dried out and cleaned; or, if circumstances permit, lagoons are unloaded into a neighboring stream at time of flood.

Observers differ as to the prevalence of odors. Apparently odors should be at a minimum when digested sludge is used. In northern climates, lagoons

fed with raw sludge undergo a dormant or refrigerating stage through the winter months. When the sludge warms up in the spring, a large volume of undigested sludge is stimulated to a degree which may create odors. The problem of odors and their effect on a local situation depends on many circumstances. Odors may be noticeable in the area occupied by the treatment works site; but, if habitations are distant, no complaint ensues. On the other hand, certain distinctive odors, such as arise in acid sludge digestion, may travel a considerable distance. Thus the type of odor is a factor.

The capacities of lagoons vary greatly, according to the use (whether for excess sludge or all the sludge) and the nature of the sludge. The smaller capacities per capita should occur with digested sludge, and the larger capacities where excess activated sludge is concerned. Considerably more capacity per capita is required in lagoons than in separate sludge digestion. The volume of lagoons receiving all the digested sludge at a plant varies from 6 cu ft per capita to 37.5 cu ft per capita. For undigested sludge, volumes from 6.87 cu ft per capita to 60 cu ft per capita are found, depending on the type of sludge.

In the southwestern states, with a monthly air temperature averaging above 60° F, open-air unheated separate digestion or lagooning should prove adaptable and efficient, with proper design for mixing and seeding and withdrawal of supernatant. The disposal of supernatant liquor from the lagoons is varied. Apparently many operators discharge the supernatant into the nearest waterway, if sufficient flow is available. Before deciding on procedure, the supernatant should be measured, sampled, and analyzed for B.O.D. and suspended solids. There is lack of reliable data on lagoons and their utilization, as well as on their location, probably because they are generally regarded as a temporary measure.

Sludge Digestion

The U. S. Public Health Service census as of 1943 (68) shows the following separate digestion plants:

Type	Heated	Unheated	Total
Municipal	780	559	1,339
Institutional	21	86	107
Total	801	645	1,446

Heat-controlled digestion was introduced at Plainfield, N. J., in 1926 and has grown in favor, particularly in the northern states by its ability to function the year around, regardless of atmospheric temperature. Multistage digestion has also become popular in two stages: The primary tank equipped with devices for heating the digesting sludge and accomplishing the major part of the digestion; and the secondary tank for storage under relatively quiescent conditions (69). At the County Sanitation Districts of Los Angeles County, Mr. Rawn uses a four-stage digestion of settled sludge. In 1947, the sludge was heated to a temperature of about 85° F in collecting tanks and transferred with about 20% of digesting sludge to the first stage, at a rate of almost 30 lb per cu ft per month. The average over all for the four stages is somewhat more than 7 lb per cu ft per month. Within a period of 10 days, 46% of the volatile

matter is digested in the four stages, whereas after 4-day digestion in one stage only 12.7% of the volatile matter is digested. The present procedure resulted from a 4-year study (70).

At New York City, in the proposed (activated sludge) Hunts Point Sewage Treatment Works, the design of sludge digestion is based on a period of about 19-day displacement in the four digestion tanks, and 26-day displacement in the four secondary digestion tanks, for an ultimate population of 961,000. Dry solids to be removed daily as sludge are assumed to aggregate 173,000 lb. The storage tank capacity is sufficient to load two sludge vessels of 55,000 cu ft each, or about three times the average daily volume of sludge to be disposed of.

Heating Devices.—Various devices are available for heating sludge, in addition to coils for circulating hot water through the tank (71). C. E. Keefer, M. ASCE, suggests (72) heating by direct contact of the circulating sludge with the hot flame and combustion gases obtained from a burner, or by using a similar burner in an enclosed chamber submerged in the sludge being heated. W. J. Wenzel, M. ASCE (73), would bury the coils in the concrete floor and walls of the tank. Heating sludge outside the digestion tanks was practiced at North Toronto, Ont., Canada, from 1929 to 1934, when it was superseded by the use of coils in the tanks. P. E. Langdon, M. ASCE (74), has proposed an external heat exchanger in which a mixture of fresh and digested sludge is circulated by a pump through pipes surrounded by hot water. This type was developed for the proposed Hunts Point plant in New York City (75) and consists of two concentric aluminum pipes, 6 in. and 8 in. in diameter. The raw sludge is pumped through the inner pipe and hot water is circulated in the opposite direction through the annular space. Mr Rawn (76), V. W. Bacon, Assoc. M. ASCE (77), and C. P. Gunson (78) heat the sludge in an outside tank with live steam before introduction into the digester. This has been practiced by Mr. Rawn at the Los Angeles County Sanitation Districts plant since 1942, and by E. V. A. Quartly, Assoc. M. ASCE (79), at San Diego, Calif., since 1945.

Scum.—Many operators (1) are troubled with both operational and maintenance scum problems in digesters. R. S. Rankin, M. ASCE (80), classifies the materials which cause scum formation in digesters as (*a*) grease and oil; (*b*) mat-forming material; (*c*) miscellaneous indigestible material; and (*d*) gas lifted sludge. Basically, there are two types of scum-forming material—digestible and nondigestible.

J. J. Gilbert (80) noted that submergence of scum was not the only answer to scum control, but that a breaker, which broke up the scum and allowed the digestible portion to sink, was sometimes effective. At the Cleveland Westerly plant the presence of paunch manure from packing houses causes trouble. W. E. Gerdel (81) describes methods of scum control, but is unable to decide which is most effective. J. Doman (82) concludes that the problem can be minimized if the temperature of the scum is at least 95° F; if the pH is slightly above 7; and if there is proper seeding—provided the scum is kept wet and thin. Undigestible material must be removed physically. Henry W. Taylor, M. ASCE, jets hot supernatant onto the forming scum blanket at Ludlow, Mass. (83). This method will be used at the new Boston plant (35).

Inhibitors of Digestion.—At Kenosha, Wis., the copper-bearing pickling and rinse water wastes from brass and copper industries interfere with the digestion of sludge (84). The toxic wastes cut the gas production rate from 11 cu ft per lb to 0.5 cu ft per lb of volatile matter. H. T. Rudgal points out that a raw sludge containing 95% water, 5% solids, and 2,500 ppm of copper actually, has about 5% copper in the dry solids. W. J. Wischmeyer and J. T. Chapman, Juniors, ASCE (85), found sludge digestion was not retarded by concentrations of nickel sulfate and nickel ammonium sulfate up to and including 500 ppm as nickel. In excess of 500 ppm, and especially of more than 1,000 ppm, digestion processes are definitely retarded.

Gas Utilization

Gas utilization at New York City began (86) in 1936 at Coney Island with three 300-hp gas engine generator sets, and circular digestion tanks 54 ft in diameter. The latest designs provide for tanks 120 ft in diameter and having side water depths of 40 ft, with multiple inlets, and provision for positive circulation. At Hunts Point, preheating of sludge before discharge to the digester is proposed. Dual-fuel engines installed in units of 460 hp at Tallmans Island are of the heavy duty slow speed type, which operates successfully on digester gas when diesel oil is injected in an amount from 5% to 7% of the heat value of the total fuel used. The engines operate with 100% oil, or on any combination of oil and gas above the lower limit. N. C. Wittwer, M. ASCE (87), discusses the principles of power generation with sewage-gas engines, with data on fourteen installations.

Gas Explosions

Gas explosions were reported at Monroe, Mich. (88), Herkimer, N. Y. (89), and Traverse City, Mich. (90). At Traverse City, gas escaping from the top of the digesters was ignited by an arcing electric switch inside an adjoining building. At Herkimer, an explosion in the sludge digester blew off the roof. An explosion (91) in an Akron, Ohio, sewer system in 1941 killed three people and injured others. In 1946, damage suits were settled for $69,842, of which the Goodrich Rubber Company paid $44,270 and the city the balance.

A gas fire (92) in the top of a 30-ft-deep manhole on a 24-in. sewer at Knoxville, Tenn., was extinguished with carbon dioxide. The gas was found to be methane seeping into the sewer from an adjacent sanitary landfill.

Sludge Force Mains

In the Third Progress Report (59), data were given on the friction losses in 1944 of the 16.2-in.-inside-diameter pipe line from the Southwest Works at Chicago to lagoons approximately 5 miles away. A comparison of the 1944 tests and others conducted in 1947 shows:

Date of test	Sludge	Solids (%)	Temperature (°F)	Hazen-Williams c
June 9, 1944	Activated	1.36	62	147
July 18, 1947	Activated and primary	1.48	73	118

The flow characteristics of the two sludges are essentially the same. The tests indicate a loss of approximately 19.5% in pipe line capacity at a given head, or about 6.5% per yr of operation. The capacity of the North Side pipe line has decreased at a rate of about 2.5% per yr.

In Cleveland, Mr. Ellms reports that the 13-mile, 12-in. force main was cleaned twice in 1946 and three times in 1947. In one of the three cleanings in 1947, the "rabbit" was put through only about one half of the length of the pipe. Cleaning of the pipe is usually undertaken when the pressure builds up to about 180 lb per sq in., when the rate of flow is about 1 mgd. After cleaning, the rate of flow is between 1.2 mgd and 1.3 mgd at a pressure of 150 lb per sq in. The concentration of the solids in the sludge may vary from 3.5% to 4.5%. The "rabbit" is leased at an annual cost of $300. The city furnishes repair parts and a trained operating crew.

Sludge Concentration

D. E. Bloodgood, Assoc. M. ASCE (93), describes sludge concentration at Indianapolis, Ind., in a tank with 3,292 sq ft of surface area. In general, the dry solids ranged in the waste activated sludge from 0.31% to 1.04%, and in the concentrated sludge from 1.09% to 2.78%. The loading ranged from 56 gal per sq ft per day to 373 gal per sq ft per day. The solids in the decanted effluent ranged from 38 ppm to 134 ppm.

Elutriation

A. L. Genter, M. ASCE, reports that no new elutriation plants have been constructed in the United States since 1945. However, in the United States and Canada, elutriation is planned in about forty-five plants serving a design population of about 14,000,000. Of these, five are to serve plants already equipped with vacuum filters handling digested sludge not elutriated. At present nine installations are being constructed.

The Richmond-Sunset plant in San Francisco, Calif., is equipped with a single-stage digester. Elutriation with plant effluent is effective in clarifying the supernatant and concentrating the solids in the combined supernatant liquor and digested sludge drawn therefrom. In enlarging the plant, a new digestion tank and an additional vacuum filter are being installed. During the major part of the day, the present elutriation tank system will serve to give the combined sludge and supernatant leaving the new primary digester a single-stage wash before the collected primary digester solids are discharged to the smaller secondary digester. During off-peak hours (the minor part of the filter operating day), the stored secondary digester sludge will be routed back through the dual elutriation tanks and given a countercurrent wash preparatory to vacuum filtration.

Vacuum Filtration of Sludge

According to the U. S. Public Health Service (68), in 1942 there were ninety-four sewage works in the United States dewatering sludge with vacuum filters, classified as follows:

Type of sludge	No.
Primary—	
Raw	15
Digested	29
Total	44
Chemical Precipitation—	
Raw	14
Digested	11
Total	25
Activated Sludge—	
Raw	10
Digested	15
Total	25

Table 1 gives the operating results for twelve installations, of which two are handling raw primary sludge, four are on digested primary sludge, and six are on elutriated digested primary sludge.

For conditioning digested sludge prior to vacuum filtration, carbide lime slurry from acetylene gas plants has proved a cheap substitute. At Wyandotte, Mich., H. M. Leonhard (94) claims an annual saving of about $5,000, with an absence of dust and less operating attention. Slurry (95) costs $1.00 per ton, containing from 2.25 lb to 3.75 lb of solid per gallon, the calcium (as CaO) varying from 71% to 75%. Water is added to make up to 2 lb of solid per gallon. At Detroit F. H. Burley, Assoc. M. ASCE (94*a*), indicates a gradual conversion to the use of carbide lime slurry, with practically no change in the dosage with ferric chloride and no noticeable effect on the filter cloth life (330 hours). Less power for equipment was required and a decided economy resulted.

Dewatering and Heat-Drying Activated Sludge

The operating costs for dewatering and heat-drying sludge have risen since 1941, because of the increased cost of labor, coal, ferric chloride, and other supplies. In the chemical industry, the postwar readjustment and increased demand for chlorine have created a temporary shortage both of chlorine and ferric chloride. In The Sanitary District of Chicago, during 1947 no ferric chloride was available for 67 days at the Calumet plant and for 74 days at the Southwest Works, necessitating a diversion of all the excess activated sludge to lagoons. Various substitutes for ferric chloride were examined, but these either proved less effective or required considerable equipment installation for temporary use. Rubber lined tank cars were also scarce. To construct new ones requires several months. Steel is difficult to obtain. Although the manufacture of ferric chloride and chlorine by a municipality is possible on a scale like that at Chicago, it appears desirable to purchase these products in the open market or under long-time contract, provided a steady supply can be assured at a reasonable price.

Incineration of Sludge

Incineration of sludge is discussed by G. J. Schroepfer, M. ASCE (96), particularly with reference to Minneapolis–St. Paul experience. He considers incineration of garbage with sewage sludge logical, and concludes that both

TABLE 1.—Performance of Vacuum Filter Installations

Location	Year	Solids in wet sludge (%)	Conditioner Used: Kind	Conditioner Used: Lb per 100 lb of dry solids	Cake: Lb of dry solids per hr per sq ft of filter area	Cake: Moisture (%)	Bibliography reference
(a) Raw Primary Sludge							
Minneapolis—St. Paul, Minn.	1945–1946	9.0	$FeCl_3$ / CaO	0.92 / 2.84	3.25	68.1	(97)
		9.14	$FeCl_3$ / CaO	1.09 / 3.24	3.31	66.9	
New Britain, Conn.	1945–1946	....	$FeCl_3$ / CaO	....	6.2	63.6	(98)
(b) Digested Primary Sludge							
Baltimore, Md.	1946	4.1	$FeSO_4Cl$	6.19	4.30	75.4	....
Buffalo, N. Y.[a,b]	1945–1946	8.8	$FeCl_3$ / CaO	2.87 / 12.63	7.4	62.5	(99)
Cleveland, Ohio Southerly plant[c]	1946	6.4	$FeCl_3$ / CaO	5.6 / 14.9	3.19	72.0	(60)
Westerly plant	1946	6.95	$FeCl_3$ / CaO	2.75 / 12.7	2.96	66.0	
Detroit, Mich., Willow Run Bomber Plant	1944	....	$FeCl_3$ / CaO	0.63 / 37.7	3.8	....	(100)
Hartford, Conn.[b]	1946	4.8	$FeCl_3$	2.29	5.66	67.6	(101)
San Diego, Calif.[b]	1945–1946 / 1946–1947	....	$FeCl_3$	6.02 / 6.31	1.52 / 1.37	74.9 / 74.8	(102) (103)
San Francisco, Calif.[b]	1944–1945	3.3	$FeCl_3$	5.02	5.2	76.3	(104)
Washington, D. C.[b]	1946–1947	7.1	$FeCl_3$	3.62	7.1	71.9	(105)
Winnipeg, Man., Canada[b]	1945	8.7	$FeCl_3$	3.4	4.9	72.3	(106)

[a] Mixture of digested primary and activated sludge. [b] Digested sludge elutriated before conditioning and filtration. [c] Mixture containing activated sludge from the Easterly works as well as digested primary solids.

flash driers and multiple-hearth installations are capable of separate or double use. However, Mr. Ellms, at the Cleveland Westerly plant, has abandoned the use of the multiple-hearth equipment as a drier to make fertilizer, because of difficulty in preventing ignition and incineration. The multiple-hearth equipment was originally developed to roast metallic ores for removal of such substances as sulfur.

Historically, sludge incineration was attempted (107) at Worcester, Mass., in 1891. The practicability of burning sludge was investigated at Philadelphia (108) in 1910, including Btu determinations. Fuel values were determined by G. M. Wisner and L. Pearse, M. ASCE (109), at Chicago in 1914. In those early days the difficulty with sludge incineration was not whether sludge would burn, but how to construct a furnace. Low-grade fuels high in ash had been burned in air suspension as early as 1901 in the United States. Handling such material (including sludge) was difficult on grates because of the masses of molten slag or ash. The intensive study of mechanical dewatering, heat-drying, and incineration carried on by W. A. Dundas and P. Harrington (110) at Chicago in 1931–1936 clarified the problem and indicated a solution.

A classification summary of heat-drying and incineration installations (up to 1948) shows:

Type of drier	Incineration	Head-dried fertilizer
Flash	9	5
Multiple-hearth	31	1
Rotary	..	2
Vacuum spray	1	.
Total	41	8

During 1946 and 1947 the only new installations for handling sludge were a seven-hearth incinerator at Dearborn, Mich., and a flash drier at Sheboygan, Wis. A combination (duo-hearth) unit for handling sludge and garbage has been installed at the Lederle Laboratories, Pearl River, N. Y., with a drying hearth over a mono-hearth refuse incinerator. The sewage sludge is taken from covered drying beds.

The spray drier at Plainfield (111) is operated with a temperature of 900° F in the drying chamber. The digested sludge is first concentrated to from 12% to 15% solids by an alum flotation process; then dried to 15% moisture. The dried material can be burned in the furnace at a value of 4,000 Btu per lb, with a production of 1,000 lb inert ash per ton. With coal at \$6.00 per ton, the drying cost is said to be \$12.13 per ton of dry solids for operation and maintenance.

Vacuator

At the Cleveland Easterly works, J. J. Wirts and A. J. Fischer, Assoc. M. ASCE (112), tested a vacuator on a pilot plant scale handling from 100 gal per min to 300 gal per min. Rates of flow ranged from 5,090 gal per sq ft per day to 15,280 gal per sq ft per day. From 3.46 cu ft per million gal to 5.98 cu ft per million gal of grit were collected, and from 936 gal per million gal to 2,234 gal per million gal of scum. From 22.6% to 40% of the suspended solids were removed. From 35% to 40% of the suspended solids and from 30% to 40% of the B.O.D. were removed in 14.5 min.

At San Diego (103) the vacuator, when operating efficiently, removed about 35% of the suspended solids. Apparently this installation is subject to rapid variation in water level, because of pump action, and thus is overloaded at times.

Density Currents in Final Settling Tanks

S. R. Kin (113) investigated the density currents in final settling tanks of an activated sludge plant. The tanks were rectangular, each 52 ft long with a surface area of 780 sq ft and with a straight line mechanism. A trough-like weir at the effluent end running back 21 ft improved results. In addition, a sheet-iron baffle parallel to the end wall and about 10 ft from it was helpful. Dropping the inlets below the surface of the sludge blanket made further improvement.

Changes at Rockville Centre Sewage Treatment Plant.—In 1946, at Rockville Centre, the performance of the final settling tanks was improved by substituting baffled center inlets for the previous multiple inlet and effluent channels.

High Capacity Filters

J. A. Montgomery (114) considers the design and operation of four types of high capacity trickling filters, in single stage with recirculation. He advises reducing the applied strength by recirculation to fit the case at hand. However, by application of the settled sewage in a raindrop spray with a filter loading of less than 2 lb of B.O.D. per cu yd of filter media and with a B.O.D. under 135 ppm, he believes an over-all B.O.D. reduction of from 78% to 85% can be obtained without recirculation. Two-stage high capacity filters have shown a 93% to 97% over-all reduction of B.O.D. without recirculation. A continuous raindrop application needs much less recirculation than a continuous film type application with heavy parallel circulation. Continuous operation of both low capacity and high capacity filters is advocated. Inferior results may be caused by excessive oil or grease in the sewage, or by return of a poor supernatant from the digester. The filter media should be free from fine material. Mr. Montgomery concludes that two-stage filters give greater reductions in the B.O.D. than a single-stage filter using the same quantity of media, and that the first-stage filter should not contain more than 65% of the total media.

TABLE 2.—Operating Results; Military Trickling Filter Plants

No. of plants	Group designation	B.O.D. load range (lb per acre-ft)	B.O.D. Removal, % Efficiency	
			Filters plus secondary settling	Over-all plant
4	Deep; no recirculation	253–429	86.9	92.7
4	Deep; recirculation	245–1,170	85.7	90.8
8	Two stage	192–930[a]	66.5[a]	90.2
4	Shallow; recirculation	705–2,080	81.7	88.1
4	Deep; recirculation	1,190–1,950	78.7	87.5
3	Deep; no recirculation	720–2,130	75.7	82.8
3	Deep; recirculation	2,520–5,750	70.3	78.3
4	Shallow: recirculation	2,370–8,250	70.1	74.8

[a] First stage.

F. W. Mohlman summarizes (115) the operation of high rate filters in army sewage works (46) in Table 2. In single-stage deep filters without recirculating

the effluent, B.O.D. ranged from 14 ppm to 49 ppm when the loading was under 300 lb per acre-ft per day. Above that loading, the B.O.D. removal dropped below 90%. Single-stage deep filters with recirculation showed B.O.D. removals greater than 90%, with a loading under 2,000 lb per acre-ft per day. In shallow filters with recirculation rates from 50% to 1,050%, the higher loadings showed poorer results. In two-stage filters, 85% to 90% over-all reductions were obtained. Mr Mohlman concludes that high rate filters produce results between primary and complete treatment with either standard filters or activated sludge.

W. A. Hardenbergh, M. ASCE (115*a*), limits the application for aerofilters to 2 lb of B.O.D. per cu yd or 3,000 lb per acre-ft per day. Critical loadings were reached at 4.5 lb per cu yd per day, when recirculation became necessary. If the applied sewage contains more than 135 ppm B.O.D., dilution is required. He emphasizes the need of adequate ventilation and drainage.

Oxidation Ponds

Oxidation ponds to attain secondary treatment of sewage first appeared in California in 1924 (7). Since 1930 some thirty installations were made, including one at a naval station. In Texas, three installations are known, including two at army camps. D. H. Caldwell, Assoc. M. ASCE (116), discusses the principles of design and the performance and operation of such ponds, and he recommends that the sewage be clarified and then detained for 25 days in ponds at least 3 ft deep, with a minimum surface area of 1 acre per 400 contributory population. Others (117) suggest from 5-day to 30-day detention. In California, a loading of around 15 lb of 5-day B.O.D. per acre-foot per day is advised, whereas in Texas (118), a loading of from 20 lb to 40 lb of B.O.D. per acre-foot per day is used. The dissolved oxygen in the pond frequently reaches 25 ppm during the day. Coliform bacteria may be reduced from 100,000 per ml to 50 per ml, or even as low as 5 per ml. By rapid and thorough mixing, odors can be controlled. Occasionally the use of sodium nitrate may be beneficial. Such sewage ponds are most adaptable to the southwestern area of the United States, where light rainfall, much sunlight, and relatively high air temperatures occur. In northern areas, where the air temperature for several months of the year may fall below 32° F, such sewage ponds do not appear suitable, unless for emergency use in the warmer months (7). Apparently where climatic conditions are favorable and land is cheap, sewage ponds may be an economical type of construction. Thus, they may have value for the smaller communities where the cost of sewage treatment must be kept at a minimum. Recently the use of oxidation ponds was recommended by Messrs. Hyde and Sullivan (45) for the Santa Clara County project, and by Messrs. Rawn and Hyde and F. Thomas, M. ASCE (44), for one of the small plants in the Orange County project.

Chemical Weed Control

To aid in controlling growths of weeds around oxidation ponds, lagoons, and other sewage structures, R. F. Goudey, M. ASCE (119), discusses handling land and aquatic weeds, and lists the weeds and chemicals suitable. For

aquatic weeds, four types of chemicals are commonly used—namely, sodium arsenite, copper sulfate, chlorine, and chlorinated hydrocarbons. The Sanitary District of Chicago has sprayed monthly with a 0.1% solution of 2–4–2 D at the rate of 100 gal per acre, to keep down weeds on the banks of lagoons.

Fly Control

In November, 1943, The Sanitary District of Chicago began operating sixteen lagoon units near Summit with a total surface area of 78 acres. Eight units, with a total surface of 97.5 acres, were added in March, 1947, making a total area of 175.5 acres. During the warmer months, at first in 1946 (for a distance of 150 ft offshore), the banks of the lagoons and adjacent surface were sprayed with a 5% solution of DDT in oil. In 1947, the spray was changed to a 2% solution of a derivative of "hexachlorbenzine," emulsified in oil or kerosene. The spray was applied by a modified power sprayer mounted on a motor truck, delivering 8 gal per min through a nozzle supplied by a spray pump working at a pressure of 600 lb per sq in. All flies and maggots were killed in the area sprayed.

L. F. Warrick and G. F. Bernauer (120) describe the use of DDT for insect control, and for filter fly control suggest an application of an emulsion to the filter to reach the larvae. A. A. Hirsch concurs (121). Care must be taken to prevent hazard to aquatic life in the receiving waters. E. J. Hansens (122) advises use of this chemical only on an experimental basis, and care in handling it.

Chlorination

The 1946 *Engineering News-Record* survey (67) shows 1,313 sewage treatment works equipped with chlorination apparatus, or an increase of 204 over the 1943 U. S. Public Health Service Census (68). Such uses of chlorine may be divided (123) into four classes: For disinfection; as an adjunct to primary treatment; as an adjunct to secondary treatment; and as an adjunct to sludge disposal. Frequently, the use of chlorine is more beneficial as a preventive than as a curative.

The current shortage (124) of liquid chlorine and chlorine containers has resulted in a scant supply for sewage chlorination. An actual lack of chlorine existed in California in the late summer of 1947, when the scarcity of electric power seriously cut chlorine production. This stopped chlorination at the Hyperion treatment plant (125), and reimposed the quarantine on 11.52 miles of ocean bathing beaches.

A "Chlorine Manual" (126) covering the proper handling of chlorine containers, measures for employee protection, and the characteristics of chlorine was published by the Chlorine Institute during 1947.

For treating wool scouring wastes, a process using calcium hypochlorite has been developed (127)(128) (see "Wool Scouring" under "Industrial Wastes").

Alkaline chlorination is known to destroy cyanides. The reactions involved and several possible procedures are reviewed by J. G. Dobson (129) who states that a minimum pH of 8.5 and a minimum of 7.35 ppm of chlorine are required to oxidize cyanides to carbon dioxide and nitrogen oxides. G. E. Barnes, M.

ASCE (130), describes the installation of such a process in a large metal-working plant for treatment of plating wastes, where it supersedes the original practice of reducing cyanide concentration by treatment with excess lime and iron coagulation. Mr. Oeming (53) approves such procedure, considering liquid chlorine and caustic soda as the preferred combination of chemicals. F. S. Friel, M. ASCE, and G. J. Wiest (131) describe the use of chlorine to remove cyanides in wastes from an aircraft factory, following the separation and removal of other toxic constituents.

A progress report (132) on the effectiveness of various forms of available chlorine in destroying certain types of microscopic organisms and viruses, indicates the importance of the acidity or the alkalinity of the medium, the chlorine concentration, and the contact periods. At Duluth, Minn., excessive alkalinity resulted from the discharge of carbide wastes into the sewage (133), causing a reduction in bacterial kill from a normal of 99.5% to 95.4%. At another sewage plant (134), a 21% increase in chlorine demand resulted from the discharge of beet cannery wastes. The red color of the wastes made the usual colorimetric residual chlorine test inapplicable. Undoubtedly more chlorine should have been applied, since the coliform index (usually below 100 per ml) increased during the canning season (to a maximum of 12,500 per ml).

At Niagara Falls, N. Y. (135), the high chlorine content of wastes discharged from certain industries reduces chlorinator operation to a part-time basis. The average residual chlorine content of plant effluent is 3.4 ppm, but varies up to 280 ppm. Residuals are determined at hourly intervals or oftener. Another problem (136) is presented by various types of effluent from the activated sludge process. The amount of chlorine required for effluent disinfection may be decreased materially when a high degree of purification is attained (because of high nitrate and low nitrite and ammonia content) as compared with the chlorine requirement of less highly purified effluent (having low nitrate and high nitrite and ammonia content).

When sulfur bacteria caused filter pooling at Camp Kilmer, N. J. (137), surface treatment with chlorine, bleaching powder, and sodium hydroxide failed. Pooling was stopped by flooding the beds and allowing anaerobic digestion to occur until all gasification ceased, taking about 2 weeks for each bed. This was followed by frequent flooding and flushing, sometimes twice daily, until heavy sloughing stopped.

N. Herda (138) found at the Willow Run Bomber Plant that, when the activated sludge plant (mechanical aeration) was overloaded and the supernatant liquid became a problem, chlorination of the return sludge and supernatant with 2 ppm to 8 ppm, based on the return sludge flow, was helpful. J. A. Tapleshay, Assoc. M. ASCE (139), suggests using chlorine to control the density of return activated sludge, with a minimum contact time of 2 min before discharge into the influent to the aeration tank. The dose is based on the sludge index and the weight of dry solids in the return sludge. J. K. Adams (140) has used chlorine on an activated sludge plant at Tenafly, N. J., since December, 1939. Recently the dose fluctuated between 3 ppm and 8 ppm, with high solids content.

For the control of sewage odors at the Richmond-Sunset plant (104) in San Francisco, 30 lb of chlorine per million gal are applied for prechlorination and 90 lb per million gal for postchlorination. Both chlorine and ferrous chloride (141) are applied to raw sewage at the San Diego treatment plant for odor control, the iron being obtained from shredded tin cans.

In a progress report on sulfide control research, R. Pomeroy and Fred D. Bowlus, Assoc. M. ASCE (142), indicate that sulfide generation is a biochemical phenomenon, caused in sewers by slimes on the submerged surface of the sewer and by deposited sludge. Sewer cleaning removes sludge deposits and chemical cleaning removes slimes. Chlorine and aeration may also be used to destroy sulfides. A pamphlet on the effect and control of hydrogen sulfide in concrete sewers (143) is issued by the Portland Cement Association.

Experiments on the use of chlorinated hydrocarbon (144)(145) have been conducted in Southern California for 8 years, with varying dosage, ranging from 1 ppm to 35 ppm "upsewer" locations, to allow a long contact time in the sewers. Apparently doses of from 1 ppm to 3 ppm will not stop sulfide generation in sewers. However, Mr. Pomeroy (in a paper read before the Los Angeles Section, ASCE, October 29, 1947) indicated that this material has been tried at the Los Angeles County Sanitation Districts in 1942–1943, by Orange County in 1942, and by Long Beach in 1946. Reliable data, when available, show no significant effect on sulfide generation in sewers, when economically practical dosages are used.

At Los Angeles, in 1946–1947, studies indicated (145) a chlorine demand in the sewage at the outfall of nearly 30 ppm, with a peak load of 92 ppm. Various chemicals were tried, including chlorinated hydrocarbon. In field tests, when applied in concentrations of from 1 ppm to 2 ppm in "upsewer" locations, R. F. Brown, Assoc. M. ASCE (144), claims the bacterial content at the outfall was reduced 50% or more, and hydrogen sulfide production along the trunk sewers was cut from 65% to 95%. In 1947, chlorination of the entire flow of sewage (average 198 mgd) consumed (146) an average of 18 tons of chlorine daily, applied by seven chlorinators, each with a capacity of 6,000 lb per day. The permit of the California State Department of Health requires such disinfection from May 15 to November 15 during the construction of the sewage treatment works. During 1947, dosage of chlorine was continuous from May 15 to October 31, except for 16 days in September. Partial chlorination was tried, using 1 ppm of chlorinated hydrocarbon with chlorine equal to 50% of the chlorine demand for one period, and 25% of the chlorine demand for another. The results of these tests are not available.

Chlorination at Buffalo (99) is effected by adjusting dosages hourly after determination of the immediate chlorine demand by the "orthotolidine test." The results in the 2 years from 1944 to 1946 are shown in Table 3.

At Cleveland (60) during the 1946 bathing season, the effluent of the Easterly plant was chlorinated for 90 days. A total of 360,000 lb of chlorine was used, equivalent to a dosage of 5.3 ppm, with a residual of 1.0 ppm of chlorine. At the Westerly plant, because of the proximity of bathing beaches, a total of 365,604 lb of chlorine was used, during a period of 107 days. A prechlorination dose of 230,309 lb, or 63% of the total, was added to the influent to reduce

odors and to prevent foaming. The additional 132,295 lb of chlorine maintained a residual in the plant effluent for 80.5% of the time. The average chlorine dosage was 17.1 ppm.

In the summer of 1947, C. W. Klassen (147) investigated the disposal of sewage in Lake County, Illinois, and its effect on the water intakes and the adjacent beaches on Lake Michigan. The survey revealed a condition of insufficient chlorination of sewage effluents from settling tanks and a lax operating control. As a result, the various authorities concerned are busy preparing for the summer of 1948.

TABLE 3.—RESULTS OF HOURLY CHLORINE DOSAGE ADJUSTMENT AT BUFFALO, N. Y., DURING 1944–1946

Year[a]	Chlorine Demand (Ppm)		Chlorine Dosage (Lb)			B. Coli (per Ml)					
						Dry Weather Flow			Entire Flow		
	Raw sewage	Over all	Average	Maximum	Minimum	Average raw	Effluent	Reduction (%)	Average raw	Effluent	Reduction (%)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1944–1945	4.6	....	4,390	7,190	2,330	....	....	....	90,900	1,830	98.0
1945–1946	4.6	4.89	4,340	12,100	1,900	87,260	1,200	98.6	80,600	1,510	98.1

[a] Year ending June 30.

Chemical Treatment

The number of treatment plants in the United States employing chemicals (other than chlorine) has apparently declined from 190 in 1943 (68) to 158 in 1946 (67). Little has been published on the use of chemicals as an aid to sewage treatment, except on chlorination. Studies on sludge dewatering, using ferric chloride, alum, and other chemicals were conducted in England, without adding to the known facts.

There is a growing interest in the use of chemicals for industrial and trade waste treatment along five lines: (1) Neutralization of acid wastes; (2) removal of caustic lime in solution (flue gas, containing carbon dioxide); (3) removal of suspended and semisolid substances; (4) removal of specific materials, particularly poisons; and (5) control of sludge lagoons or digestion tanks (nitrates). In the first group belongs the use of lime in various forms, of soda ash, and of caustic soda, to neutralize acid wastes which may cause sewer corrosion and deterioration of equipment, or to affect biological treatment devices. In the second group is a method for removing caustic lime from industrial wastes by the use of flue gas developed by H. W. Clark, G. O. Adams, and A. C. Bolde (148). At one time in the South Essex (Mass.) Sewage Board jurisdiction, nine carbonating plants were in operation. Flue gas contains from 7% to 12% carbon dioxide, which combines with the calcium hydroxide in the waste and forms calcium carbonate, which readily precipitates out and is settled before discharge to the sewer. Professor Camp (40*a*) has recommended such pretreatment in the Merrimack Valley project.

The third group is concerned with clarification and comparatively low B.O.D. removal. Wastes containing organic suspended solids produced by the food industries are treated with lime or sulfate of alumina, or both. Well-known processes of lime precipitation have been tested on canning wastes. Various methods for handling wool scouring wastes have been used for more than 40 years, including acid cracking. However, most of the effluents produced have required further treatment before discharge. The most recent development is a process patented by J. L. Campanella (128) for treating wool scouring wastes, using calcium hypochlorite in large amounts.

In the fourth group, chemicals are used for the removal of specific substances in the waste, such as cyanides and cyanide compounds, which are used in industry, particularly in the plating of metal, the case-hardening of steel, the neutralizing of acid "pickle scum," the refining of gold and silver ores, the scrubbing of gases from blast and producer gas furnaces, and the making of certain pharmaceuticals.

The fifth group relates to the effluents from the processing of vegetables and fruits (59).

Apparently large producers of ferric chloride are unwilling as yet to accept long-term contracts for specified deliveries. As a result, small organizations have sensed an opportunity and offer the material at a high price, gambling on the chance that an occasional tank car of chlorine and enough scrap iron to make a delivery can be picked up.

In sewage coagulation, normally ferric chloride could be replaced by sulfate of alumina or ferric sulfate or chlorinated copperas. If chlorine is not readily available at a reasonable price, chlorinated copperas cannot be relied upon. With the substitutes, feeding devices may need revision, a longer period may be required for coagulation and settling, and the degree of purification may be less if the additional coagulating and settling capacity is not available.

The chemicals used in coagulation of sewage are primarily lime, ferric chloride, ferric sulfate, chlorinated copperas, aluminum sulfate, or combinations. In 1946–1947, there was a scarcity of lime, ferric chloride, chlorine, hydrochloric acid, and sulfuric acid. Furthermore, costs have risen. A shortage of tank cars for transportation of chlorine, acids, and ferric chloride (rubber lined) has interfered with obtaining raw materials and the delivery of the product, such as ferric chloride. The problem has been complicated by readjustments of large chemical companies—such as discontinuing the manufacture of ferric chloride in the north and shifting to a locality more than 1,000 miles away, with cessation of the production of ferric chloride.

For conditioning sludge for vacuum filtration, and, particularly with activated sludge, ferric chloride is still the most effective chemical. Less effective are ferric sulfate, chlorinated copperas, and sulfate of alumina. Any change in an existing installation based on the use of ferric chloride may result in difficulties, with lowered efficiency and probably with higher costs.

The costs of chemicals used for conditioning digested sludge at the Cleveland Southerly works (60) during the years 1942 to 1947 were as follows: (The second and third columns represent cost per ton; and the fourth and fifth columns, cost per ton of dry solids.)

Year	Ferric chloride	Lime	Ferric chloride	Lime
1942..........	$30.72	$ 9.19	$1.33	$1.54
1943..........	30.80	8.80	1.29	1.47
1944..........	30.80	9.79	1.53	1.77
1945..........	30.80	9.80	1.45	1.62
1946..........	38.69	10.10	1.64	2.15
1947..........	47.63	12.00	2.85	1.39

Price data on the cost of liquid chlorine and bleaching powder are given in Table 4. The costs of ferric chloride, and of Illinois coal (approximately 11,500 Btu per lb as fired), f.o.b. Chicago, are given in Table 5.

TABLE 4.—Cost, per 100 Lb, of Liquid Chlorine and Bleaching Powder[a]

Year[b]	Liquid chlorine[c]	Bleaching powder[d]	Year[b]	Liquid chlorine[c]	Bleaching powder[d]
1921	$6.00	$2.25	1935	$2.00	$1.90
1923	4.00	1.35	1937	2.15	2.00
1925	4.00	2.00	1939	1.75	2.00
1927	3.50	2.00	1941	1.75	....
1929	2.75	2.25	1943	1.75	....
1931	1.75	2.00	1945	1.75	....
1933	1.85	1.90	1947	2.00	....

[a] Prices from United States Department of Labor. [b] As of November 15. [c] Tank cars. [d] Car lots.

The outlook for adequate supplies of chlorine and ferric chloride for sewage treatment and sludge conditioning has improved markedly since the spring of 1947. Large producers are beginning to seek long-term contracts; small producers, in seeking to establish a footing, will accept odd lot orders.

TABLE 5.—Cost of Ferric Chloride and Illinois Coal at Chicago

Year	Ferric chloride, per 100 lb[a]	Coal, per ton[b]	Year	Ferric chloride, per 100 lb[a]	Coal, per ton[b]
1932	$1.724	....	1941	$1.529	$3.39
1933	1.991	....	1942	1.526	3.35
1936	2.081	....	1943	1.505	3.70
1937	2.258	....	1944	1.505	3.92
1938	2.261	....	1945	1.505	4.04
1939	1.532	....	1946	1.505	4.41
1940	1.529	$3.19	1947	2.836 to 5.00	5.24[c] 6.19[d] 7.00[e]

[a] Cost per 100-lb of ferric chloride (dry weight) plus water, delivered as solution containing 41% ferric chloride. [b] Approximately 11,500 Btu per lb as fixed. [c] January 1 to June 1. [d] June 1 to October 31. [e] November–December.

According to data from the United States Bureau of the Census, the amount of chlorine produced yearly in the United States rose from 37,985 tons in 1921 to 446,260 tons in 1937. Actual tonnage may have been around 10% higher in 1937. By 1942, chlorine production had risen to 1,065,775 tons; and by 1944, to 1,343,950 tons. In 1947, the estimated production was 1,383,290 tons. Some additional capacity is under construction. These data indicate a re-

markable growth from 1921 to 1947, and a prospect of sustained output, with possibility of increase as the demand arises, subject to the time required to construct new capacity.

Industrial Wastes

Various industrial wastes are discussed by the Committee on Research of the Federation of Sewage Works Associations in its review of the literature of 1945 and 1946 (1)(149), and by a symposium of the American Chemical Society (150). A brief guide covering some twenty industries (151), was issued by the Interstate Commission on the Potomac River Basin, for use by the general public. The conferences on industrial wastes at Purdue University, Lafayette, Ind. (50)(152), have also developed much useful information.

At its River Rouge plant in Dearborn, the Ford Motor Company (153) maintains a waste control program built around a special water and waste laboratory, which not only directs the operation of the company's water purification and sewage treatment plants, and controls the water supplies and swimming pools of the company, but also studies wastes, initiates research, and maintains technical relations with governmental agencies on phases of sanitary engineering and public health relating to water supply, sewage disposal, and industrial wastes of the company.

In view of the present availability of the many references on industrial wastes, hereinafter cited, the Committee has omitted brief abstracts which might properly be made. Among the wastes most frequently discussed are those from the following industries:

Industry	Bibliographical reference
Acid mine drainage	(154)
Beet sugar	(155)
Brass and copper	(156)
Brewery	(50*b*)
Canning	(50*c*)(157)(158)
Citrus canning	(159)
Chemical	(160)
Coke and gas	(161)(162)
Coal mining	(154)(163)
Corn starch	(164)
Dairy	(165)
Fermentation	(166)
Metal plating	(50*d*)(50*e*)(129)(130) (167)(168)(169)(170)
Meat packing	(49)(50*a*)(171)
Petroleum refining	(50*f*)(172)
Pharmaceutical and biological	(173)
Pulp and paper	(50*g*)(53)(174)
Paper de-inking	(175)
Spent pickling liquor	(176)(177)
Rubber	(178)
Tanning	(179)
Textile	(180)

Wool Scouring.—Wool scouring wastes are among the strongest wastes produced by the textile industry and contain a valuable wool grease. Although these wastes have been a source of trouble in Great Britain since 1868, it was not until 1902 that Bradford installed a process to treat the mixed sewage and wastes and recover the grease. From 1909 to 1938, the development has been outlined by H. Stabler and G. H. Pratt (181), H. R. Crohurst and A. D. Weston, M. ASCE (182), and the late R. S. Weston, M. ASCE (183)(184)(185), along the lines of acid cracking and, since 1917, the use of centrifugals. Calcium hypochlorite (sometimes called "chloride of lime" or "bleaching powder") was early used in sewage treatment (186)(187). In 1910, H. W. Clark (188) experimented on wool scouring liquor, using from 10,000 lb to 20,000 lb of bleaching powder per million gal of waste. In 1925, A. J. deRaeve (189)(190) developed a process for lime precipitation, followed by chlorination and acidification. Recently, Mr. Campanella has patented a process (128) for applying chlorine as calcium hypochlorite solution, instead of gaseous chlorine, in a continuous mixing and aerating unit, followed by settling. H. A. Faber (127) states that the test plant removed about 90% of the suspended solids, B.O.D., and grease. The effluent may contain from 25 ppm to 50 ppm residual chlorine. The results cited indicate as much as 33,300 lb of chlorine may be required per million gal of waste, as well as 40,000 lb of sulfuric acid for handling the sludge and the scum. The economy of the process apparently depends on the amount of grease recovered and its salability.

Effect of Industrial Wastes on Sewage Works Operation

Among the industrial wastes which affect the operation of sewage treatment works are the following:

Brewery Wastes.—At the Irwin Creek activated sludge plant at Charlotte, N. C. (191), brewery yeast clogged diffuser plates, increased the ferric chloride requirements for sludge filtration, and increased the air required. The brewery was required to remove the solids and to treat at the source barrel washing wastes which proved troublesome. At Cranston, R. I. (192), the heavy industrial load, especially from a brewery and print works, prevented operation of an activated sludge plant.

Chromium Wastes.—At Tallmans Island activated sludge plant in New York City (193) a 30-min discharge of chromate occurred, totaling more than 2 tons of chromium, with a concentration in the sewage of 430 ppm as chromium. This was followed by a second dose of 2-min duration the next morning, which resulted in 1,440 ppm chromium and a reduction of pH to 4.2. Production of nitrites stopped for 10 days; and that of nitrates, for 13 days. The suspended solids increased from 7 ppm to 31 ppm and then decreased for 10 days to 6 ppm. The B.O.D. values in the effluent increased from 15 ppm to 20 ppm and then decreased to 12 ppm in 3 or 4 days. No deleterious effects on the settling digestibility of the sludge were noted.

Formaldehyde.—At the Ley Creek Works, near Syracuse, N. Y., an activated sludge plant (194), the discharge of formaldehyde wastes from a penicillin plant stopped digestion. Formaldehyde was used to sterilize the vats and to

hold the medium to prevent bacterial growths. In laboratory tests, 25 ppm of formaldehyde delayed digestion for 10 days; 50 ppm, for 20 days; and 100 ppm stopped digestion altogether.

Acetylene Manufacturing Wastes.—At Duluth, Minn. (133), three occurrences of industrial wastes from an acetylene manufacturing plant increased the pH of the sewage from 7.34 to 7.69 to 8.40 to 8.45. The normal bicarbonate alkalinity increased from 160 ppm to a total of 268 ppm. The bacterial count in the raw sewage decreased, but increased in the plant effluent to 19,621 per ml over an average of 3,409 per ml. The effectiveness of chlorination decreased although pH and gas production of the sludge digesters remained normal.

Atomic Energy

In certain localities, the development of atomic bomb research has caused anxiety because of possible wastes and effect of the procedure on the cooling water used. For those interested, two general references are of value (195) (196). A. C. Klein (197) describes atomic bomb engineering in general terms, and more specifically the engineering features of the atomic plants (198). The water problem is discussed by I. Perlman (199) and H. M. Parker, Assoc. M. ASCE (200), reviews water monitoring procedures at the Clinton laboratories near Oak Ridge, Tenn. W. F. Bale (201) outlines health protection in the production and the use of atomic energy. In general, cooling water as such seems likely to be harmless. However, certain solid residues occurring in small amounts apparently may be dangerous and should be buried in a suitable spot.

Sewage Effluents for Industrial Use

Recent experience with the salvage and the sale of municipal sewage effluents for industrial purposes is summarized by N. T. Veatch, M. ASCE (202). In many cases the cost was less than that for the development of an equal volume of additional water. Unless unusual amounts of soluble mineral salts are involved, the additional treatment of effluent for industrial use is neither difficult nor expensive. Ten places are cited, principally in the west and southwest. At Baltimore trickling filter and activated sludge effluents are purchased by the Bethlehem Steel Company. The trickling filter effluent is given supplemental treatment by chemical precipitation (alum dosage of 4 g or 5 g per gal, 42-min flocculation, and 1.5-hour settling). In 1946, the cost of this industrial water was 1.73¢ per 1,000 gal, exclusive of interest and amortization of investment. Under an agreement with the City of Baltimore, the company pays monthly for the effluent used, according to the following schedule:

Amount (mgd)	Monthly payment
Less than 25	$1,000
25 to 37.5	1,500
37.5 to 50	2,000
50 to 62.5	2,500
62.5 to 75.0	3,000
75 to 87.5	3,500
87.5 to 100	4,000

In 1947, approximately 25 mgd was sold, of which 10 mgd was activated sludge effluent. In 1948, probably 50 mgd will be used.

In Los Angeles County, California, Mr. Goudey (203) suggests the return of chlorinated activated sludge effluent to the ground water for general water use in a system including 110 effluent wells (16-in. casing, 400 ft deep) about 2,000 ft apart and not less than 1,000 ft from any wells used for water supply. He considers such rectified sewage effluent would be better for irrigation and industrial use than Colorado River water.

On the other hand, Messrs. Rawn, Hyde, and Thomas (44) conclude, for the Orange County project, that the excessive costs involved in producing an acceptable effluent and the limited availability of spreading ground with respect to both area and period of use, together with the hazard of introducing boron into the underground water, combine to make the use of reclaimed sewage impracticable by the method of percolation. The cost of reclaiming sewage would be about twice the cost of purchasing fresh water. At some points (204) in Los Angeles County where no sewers exist underground water has been polluted from the disposal of industrial wastes in leaching pits.

The subject of pollution of underground water is now under investigation by an Assembly interim committee of the California State Legislature. In the California Health and Safety Code, provisions relating to sewers and sewer systems provide (Section 5413) that "no person shall maintain a sewer well or sewer farm without a permit" and (Section 5439) that no permit shall be granted for a sewer well extending into a usable water stratum. Furthermore, the state board of health "disapproves of the practice of disposing of road or land drainage into wells reaching to water strata used or suitable for domestic well water."

Cooling Water Benefits from Increased River Flows

Mr. LeBosquet (205) describes an investigation for the U. S. Public Health Service on the effect on the stream pollution problem of heat introduced by the use of the stream for cooling or condensing purposes. His analysis is based on an observation (206) that a mill pond may lose heat from its water surface at the rate of 4 Btu per sq ft of surface area per hour per degree Fahrenheit of difference in the temperature between water and air. Mr. LeBosquet concludes that an increased stream flow may prevent recirculation and deliver a cooler water downstream. However, the general problem is not so simple. The loss of heat from the water surface of a stream depends on various factors, including the rates of evaporation and convection, the humidity, wind movement, difference of temperature between the air and the water, and velocity of travel of the stream. Edward A. Birge (207) observed an average fall of water temperature during November in Lake Mendota, Wisconsin, from 52.5° F to 39° F. The average air temperature was 36° F. The average heat loss per square foot of surface per day was about 1,125 Btu, or about 115 Btu per sq ft per day per degree of temperature difference between the air and the water. Others (208) in early December found a loss in Btu per square foot of surface per degree difference of temperature per day ranging from 83 to 121.7 and heat losses (209) from the surface of Lake Michigan in the winter of 1941–1942 ranging from a high

of 1,698 Btu per sq ft per day in December–January to a low of 300 in the middle of March. J. G. G. Kerry, M. ASCE (210), calculates heat losses on the open Saint Lawrence River ranging from 633 to 1,376 Btu per sq ft per day.

Disposal of Sludge and Wash Water from Water Filtration Plants

According to F. O. A. Almquist (211), the disposal of sludge from settling basins and wash water from six rapid sand filter plants in Connecticut is developing a nuisance. In some cases, lagoons have been tried. He suggests discharge into the municipal sewer system, if practicable, otherwise, controlled discharge to a large enough stream, sludge drying beds, and lagooning. In the cities bordering the Great Lakes, the wash water and sludge usually are discharged into the lake. Inland cities with water softening plants create a problem for the sewage works. At the Hinsdale (Ill.) Sanitary District sewage treatment works, the lime sludge from the village water softening plant introduced from three to four times as much solid as the sewage solids and interfered with digestion in the Imhoff tanks. In a preliminary committee report of the American Water Works Association on Disposal of Wastes from Water Purification and Softening Plants, W. W. Aultman, M. ASCE (212), states that lime and lime-soda sludge in 1947 were commonly lagooned, although discharge into a watercourse was also practiced. However, the tendency is to eliminate such pollution. He believes that dewatering and drying are also possible, as well as dewatering and calcining to recover lime for reuse. Brine disposal from sodium zeolite softeners may best be handled (213) by controlled dilution. In some cases large slugs of brine in sewers have upset treatment works, or killed fish in rivers. A high salt content in a stream may be injurious to chickens, hogs, sheep, or cattle. In general the method adopted (214) for disposal of wash water from filtration plants must depend on local circumstances.

Dual Disposal of Garbage with Sewage

There are three methods of dual disposal of garbage with sewage (215): (*a*) By installing household garbage grinders and discharging the ground material mixed with water into the sewer; (*b*) by installing central municipally operated stations for grinding garbage, which is hauled, ground, and dumped into the sewer; and (*c*) by hauling garbage to the sewage treatment works, where it is ground and discharged either into the raw sewage or into the digestion tanks. The installation of household grinders apparently has progressed slowly from its inception around 1921. Until the beginning of World War II, only some 55,000 installations were made. A number of manufacturers now offer household grinders, so their use may become more general. Where municipal collection of garbage is available for a low charge (or included in the general tax rate), the cost of the household grinder deters its use. From the municipal standpoint, it is difficult to force every householder to buy an individual grinder. Hence, the municipality still has to maintain garbage collection and disposal. Furthermore, the thinking householder may often find that the total annual cost of a household grinder (including interest, amortization, and repairs) exceeds the charge for collection made by the municipality, if any.

Thus, the installation of a household grinder may appeal chiefly in isolated locations where garbage is not municipally collected.

Grinding garbage and dumping it into a trunk sewer, as practiced at St. Louis, does not appear to be a permanent solution of the garbage problem, even though a considerable volume of flow is available in the Mississippi. Eventually some form of sewage treatment may be required.

Where garbage is hauled to the treatment plant, ground, and discharged into digestion tanks, ample digestion space must be provided. With heat controlled, separate digestion, W. R. Drury, M. ASCE (as described in a letter), provides a digestion tank volume of 5.65 cu ft per capita at Port Huron, Mich., on a weak sewage, and 9.1 cu ft per capita at Ann Arbor. At Lansing, Mich., the planned enlargements are based on 8 cu ft per capita. The present plant has insufficient digester capacity with about 3.75 cu ft per capita. S. L. Tolman, M. ASCE (216), believes that, when garbage is added at the rate of 2 tons per million gal of sewage, the digester capacity required for primary solids plus garbage is 5 cu ft per capita; and, for primary and secondary solids plus garbage 9 cu ft per capita. E. H. Barton (216*a*) concurs on 9 cu ft per capita and advises that with the activated sludge process not more than 4 tons of garbage should be added to 1,000,000 gal of sewage. In the smaller towns, where garbage can be ground at the sewage works, the general opinion is that the ground garbage should be kept separate from the sewage, and introduced into the digester.

There are several reasons why cities are failing to adopt the dual disposal of garbage and sewage. One is that the garbage must be separated from the rubbish, preferably in each kitchen. In Chicago, for years only a single collection of mixed refuse and garbage has been made. Many families do not even own a single garbage can. To force separation on more than 550,000 families is a difficult task. A second reason may be the type of sewage treatment. Garbage as collected averages about 80% moisture. If ground and mixed with other sludge, from sedimentation, the mixed material may contain from around 94% to 96% moisture. Reducing this moisture by dewatering equipment entails expense.

On a municipal scale, the problem is largely one of economics and the type of existing sewage treatment works. In any case, the garbage has to be separated and collected. Once loaded in trucks, it can readily be transported to a suitable site, for disposal by incineration or reduction, in apparatus entirely separately from the sewage treatment works. There should be no objection to placing the garbage works on the same general site with the sewage treatment works, and under the same management.

Mr. Drury states (in an address on "Disposal of Garbage with Sewage" before the School of Public Health of the University of Michigan at Ann Arbor in 1947), that garbage must be clean and free from rubbish when handled at a sewage works. The quantity may vary from 0.50 lb per capita to 0.75 lb per capita, wet. At Lansing, in 1946, 0.75 lb of wet garbage was collected per capita daily, containing 18.8% solids. The dry solids averaged 0.14 lb per capita per day, of which 88.7% was volatile. At Ann Arbor in 1944–1945, the garbage averaged 0.48 lb per capita per day, wet. There is also a seasonal

variation with ratios between the maximum and average month at Lansing of 147%; and, at Ann Arbor, of 171%. The maximum occurred in September. To separate the bones, egg shells, glass, metals, and other solids, receiving tanks are provided, in which the mass of ground garbage is aerated and settled. The upper volume of the tank is decanted and the settled rubbish removed at the bottom.

In Michigan, Lansing is still handling sewage with garbage and the capacity of the plant is to be doubled. Midland is handling garbage in a plant designed for sewage treatment only. The Ann Arbor sewage treatment works is being remodeled and enlarged to handle garbage. Port Huron is constructing a plant for primary settling and chlorination, with separate digestion of ground garbage and sewage sludge.

W. Rudolfs, M. ASCE (217), indicates the effect of ground garbage on various types of sewage treatment. Mr. Tolman (216) estimates in general that when garbage is added to raw sewage the suspended solids are increased from 25% to 35% and the B.O.D. is increased from 18% to 26%, depending on the solids in the raw garbage. The increase in strength of primary settled sewage after 2-hour settling will be from approximately 10% to 14% in suspended solids and from 11% to 16% in B.O.D.

Use of Sludge as Fertilizing Material

Since June, 1946, Milwaukee has been selling its entire production of heat-dried activated sludge in bags, the demand still being greater than the supply. In bulk, the 1947 price was about $4.15 per unit of ammonia and $0.40 per unit of available phosphoric acid, f.o.b. cars at the sewage treatment works in Chicago. However, the commercial outlook for organic nitrogenous material is still uncertain. During World War II, the price was controlled by the Office of Price Administration ceiling. Since the removal of ceilings, the price has risen for tankage, fish scrap, and other organics, to a greater degree than for heat-dried activated sludge. At San Diego, a relatively low grade heat-dried digested sludge was sold since July 1, 1946, under a 2-year contract, for $20.00 per ton, in 100-lb sacks, f.o.b. the plant. The contractor supplies the sacks. During the year ending June 30, 1946, 1,249 tons were produced and disposed of.

In 1946, *Manual No. 2* of the Federation of Sewage Works Associations (218) appeared, describing the utilization of sewage sludge as fertilizer, with factual material of interest to all workers in the sewage field. However, cost data given therein should be carefully scrutinized, because of changes in the economic cycle.

The Minneapolis–Saint Paul Sanitary District reports that, during the fall and winter season of 1946–1947, the issuance of fresh filter cake was suspended. The Minnesota Department of Health will approve the use of this material as fertilizer providing it is stored or composted for a period of at least 6 months and provided it is not used for fertilizing vegetables that are consumed uncooked. It may be used if heat dried. However, for 8 years, fresh cake was issued to farmers during the fall and winter only, and no diseases were ever traced to it.

F. E. Bear (219) indicates that manures and composts provide only a temporary answer to the minor element problem, and that neither manure nor chemical fertilizer will necessarily deliver to the plants all the mineral elements required by the animals and men that consume such plants. W. Thomas (220) discusses composts, manures, and inorganic fertilizers, their function, use, and effect on soils and plants, and concludes that the sound view is to use organic materials of all kinds for humus, and chemical fertilizers for the additional nutrients. With this, E. Truog agrees (221).

Those interested in trace elements, micro-nutrients, and minor elements, will appreciate a digest by W. Stiles (222) of the methods of investigation for trace-element deficiency diseases of plants, and the functions of trace elements in plants and in animals.

Effect of Sewage Effluents on Aquatic Plants

In certain seaside localities a nuisance, at times of considerable magnitude, was produced (223) by the decomposition of masses of green seaweed, chiefly of the species *Ulva latissima*, but also of certain varieties of *Enteromorpha*, such as *E. compressa* and *E. intestinales*. The indications are that the growth of these seaweeds in quantity, and especially that of the *Ulva*, may be traced to sewage pollution in the waters where they are found. For a number of years a most serious nuisance of this kind occurred in the upper reaches of Belfast (Ireland) Lough during the summer and autumn. The stench at low tide was intense accompanied by sulfureted hydrogen odors.

Complaints of the conditions of small streams receiving well nitrified effluents of sewage works occurred in Massachusetts prior to 1916 at Marlboro, where the effluent of intermittent sand filters discharged into a small brook (107*a*). The growths in the stream receiving the effluent from the Brockton sewage treatment works were described by R. S. Weston and C. E. Turner (224) in 1917. A. L. Fales and E. S. Chase, Members, ASCE, canoed downstream in September, 1926, and found that masses of decaying growths were more luxuriant than in 1917 and gave off very offensive odors.

In 1925 (225) the Committee on Sewage Disposal, Engineering Board Review of the Sanitary District of Chicago, commenting on the future conditions in the Illinois Waterway with a low diversion from Lake Michigan and a greater volume of sewage effluent stated:

> "With such a concentrated effluent, the nitrate content would be relatively high, and this would stimulate plant growth in the stream which, upon decay, might of itself cause objectionable conditions."

In a letter the late H. W. Clark in 1926 stated that the principal nuisances of this type in Massachusetts occurred from the growth of weeds and blue-green algae in ponds fed by ground water high in nitrates, which were derived from a rich market garden region where the ground was highly manured or from an unsewered town where cesspools were common and the leaching therefrom became high in nitrates before entering the ponds. In Spy Pond in Arlington, vigorous growths of waterweeds and algae were stimulated by waters supposedly rich in nitrates. Near Hanson, the effluent of a sewage filter

stimulated algal growths in a small pond, which subsequently decayed and produced odors.

Stream conditions in the Back River, which receives effluent from the Back River sewage works of Baltimore, have been a source of complaint for many years from residents in the vicinity, according to G. L. Hall (226). Algal growths several inches thick extend from 20 ft to 30 ft offshore in warm weather, promoted by the high nitrogen and carbon dioxide contents of the works effluent. Neither chlorination of the effluent nor copper sulfate treatment of Back River were effective. Because of the limited volume of diluting water available, the removal of the effluent would be helpful.

Problem of Nitrogen in Sewage Disposal

The problem of sewage disposal in some instances appears to be entering a new phase, in which attention is shifting from B.O.D. and oxygen balances in their usual sense to one of fertilization, increased biological productivity of receiving waters, and attendant problems. In the case of lakes, as well as of rivers, this stimulation may result in serious nuisance conditions. C. N. Sawyer (227) believes the ammonia nitrogen is more important than nitrate nitrogen, as a stimulant to explosive algal growths, and that phosphorus is a key element in determining biological activity in a body of water. Most of the lakes around Madison, Wis., have average concentrations of phosphorus in excess of 0.10 ppm. However, nuisance conditions can be expected with a concentration of inorganic phosphorus in excess of 0.01 ppm. The critical level for inorganic nitrogen appears to be 0.30 ppm.

Even after sewage has been treated to a high degree (such as from 92% to 95% removal of B.O.D.) by the activated sludge process, it is impossible to remove certain organic solids dissolved in the sewage or in the effluent. The inert solids remaining are not harmful, but the carbonaceous and the nitrogenous compounds remaining in the treated sewage effluent may be a potential source of trouble unless the waterway into which they are discharged has sufficient volume to prevent deteriorating biochemical reactions and growth of plants.

The removal of the nitrogenous compounds is a more difficult matter than the removal of B.O.D. No matter how completely the sewage may be treated and the nitrogenous compounds oxidized, there is no known feasible method of reducing the nitrogen in the sewage by more than 50% or 55%. In the near future the nitrogen residue in the sewage effluent of the works of the Sanitary District of Chicago may be 49% of the original sewage content, or around 43 tons per day. This can act as a fertilizer in the waterway, to stimulate growths, which ultimately will decay. The intensity of the transformation of nitrogen depends on the concentration present. The required phosphorus is practically always present in sewage effluents. In 1945, the four plants of The Sanitary District of Chicago received 87.7 tons of nitrogen daily in the raw sewage. As then operated, the effluents contained 64.3 tons of nitrogen daily, consisting chiefly of dissolved nitrogen, such as ammonia, nitrites, and nitrates. At Lockport, Ill., the terminus of the Main Channel, the nitrogen content in the

flow was around 8 ppm. In the future it may be around 5.2 ppm; Lake Michigan only contains 0.1 ppm.

The U. S. Public Health Service (228) reports the nitrogen content in the Ohio River below Cincinnati, Ohio, and Louisville, Ky., as 1.12 ppm and 1.32 ppm, respectively; in the Mississippi River at New Orleans, La., as 0.86 ppm; in the Potomac River below Washington, D. C., as 0.45 ppm; and in the Licking River, at Latonia, Ky., as 1.57 ppm.

The nitrogen content of the Madison lakes is reported only as inorganic nitrogen (229). This varies from winter to summer, with a summer minimum of 0.06 ppm, and a winter maximum in one lake (Waubesa) of 2.49 ppm. In three other lakes in Wisconsin, relatively unpolluted, 0.05 ppm is reported for a summer minimum in one lake; and 0.87 ppm, as a winter maximum in another lake. Residual nitrogen is considered an index of pollution and dilution, and apparently can be reduced only by dilution. The concentration is a measure of degree of pollution and the degree of stimulation of unwanted growths. Hence, in studying pollution problems in critical situations the nitrogen content must be considered as well as the oxygen balance. Observations are desirable to determine the total nitrogen content that can be carried without promoting growths of water plants and weeds, and the onset of unusual bad odors and foul conditions.

C. H. Mortimer (230) points out that, although nitrogen as gas makes up four fifths of the atmosphere, plants can only utilize it in the form of an inorganic salt, usually nitrate. If nitrogen gas is present only in small amounts, organic production is limited. Nitrate may be produced by nitrification in the soil and washed out. It may also be supplied by the decomposition and mineralization of organic matter in the mud at the bottom of a lake. Mr. Mortimer holds there is no known process for eliminating nitrates from water. The presence of phosphorus is also important. Among the English lakes, low organic production occurs with nitrates from 0.06 ppm to 0.10 ppm; and total nitrogen, from 0.18 ppm to 0.51 ppm. Higher organic production occurs with nitrates from 0.29 ppm to 0.36 ppm and total nitrogen from 0.62 ppm to 1.16 ppm.

Problem of Phosphorus in Sewage Disposal

Phosphorus is another element that must be considered, as Mr. Sawyer (227) noted that nuisance conditions can be expected with a concentration in excess of 0.01 ppm. Apparently there is no known feasible method for removing the phosphorus. Under tropical conditions, A. Howard (231) suggested cultivating water hyacinth to recover soluble salts of nitrogen and phosphorus, using the plant in making compost.

Fertilization of Lakes

At Madison (227), agricultural and urban drainage has fertilized the lakes and stimulated heavy growths of algal blooms. An effort was made to control the growths by the use of copper sulfate, over a combined area of 13.5 sq miles.

> "For Lake Waubesa, whose inflowing waters consist of approximately 15% biologically treated sewage, 75.3% of its inorganic nitrogen and 87.6% of its inorganic phosphorus were contributed by the effluent from the Madison sewage treatment works."

The lakes downstream from the city were found to be fertilized with nitrogen in amounts ranging from 127 lb per acre per yr to 588 lb per acre per yr, and by phosphorus ranging from 19 lb per acre per yr to 88.6 lb per acre per yr. Two lakes contained 0.25 ppm of phosphorus at all times. The Madison lakes retained from 30.4% to 60.5% of the nitrogen received.

Loss of Sanitary Sewage Through Storm Water Overflows

In the design of intercepting sewers, connected to combined sewerage systems, engineers endeavor to determine how much storm water runoff will be intercepted. The proportion of storm overflow and the frequency of overflow are usually matters of speculation. Few data based on actual observations or on an analysis of the problem are available.

In analyzing the problem using statistics at Boston for the years 1934 to 1945, inclusive, J. E. McKee, Assoc. M. ASCE (232), found that rainfall equal to, or in excess of, 0.01 in. per hr, occurred 6.6% of the total time, whereas precipitation recorded as a trace or more occurred 14.9% of the total time. Storm runoff equal to the dry weather sanitary flow is produced by a rainfall of 0.01 in. per hr after impervious surfaces are well wetted. When twice the average dry weather flow is intercepted, about 2.7% of all the domestic sewage may escape, or 97.3% of all sewage will be intercepted during the months of June through November. If the interceptor is designed to take five times the dry weather flow (that is, 0.04 in. per hr of storm water), the percentage of sanitary sewage escaping can be reduced to 1.2%. The frequency of overflow with twice the dry weather flow intercepted is about five times to six times per month in the summer. With ten times dry weather flow intercepted, the number of overflows may be reduced to three per month.

Activated Sludge Patent Suit

The suit against The Sanitary District of Chicago for infringement of patents on the activated sludge process, instituted in the United States District Court in 1924, was tried before Judge Walter C. Lindley, who found that the patents were infringed but denied an injunction because of the health hazard (October 18, 1934). A rehearing was denied and an interlocutory decree was rendered against the Sanitary District. The district appealed to the United States Circuit Court of Appeals, which upheld the lower court. Thereupon, the district filed a petition with the United States Supreme Court for a review of the case, which was denied. Similar suits against Milwaukee resulted in judgments amounting to $4,707,402, which were settled for $818,000.

In the Sanitary District case, in 1939 new evidence came to light pertaining to the right of plaintiffs to sue and maintain their suit upon the patents. The Sanitary District petitioned for leave to reopen the case. This petition was denied by the United States District Court, the Circuit Court of Appeals, and the United States Supreme Court.

Pursuant to court orders, the Sanitary District filed: (*a*) On January 19, 1943, a full itemized statement of all costs and expenses incurred by it in building and acquiring each of its sewage treatment works between 1919 and

the present time, including the dates of payment and the identity of the contract or contracts under which the expense was incurred; and (*b*) on April 26, 1943, a year-by-year statement of all costs and expenses incurred by it in the operation of each of its three sewage treatment plants held to have infringed the process patents, excluding therefrom all depreciation and interest costs, and also giving a year-by-year statement of the population and the estimated population equivalent of trade wastes served by each of the three alleged infringing plants.

The hearing on the accounting was held before Judge Lindley from October 29 to November 2, 1945, to determine a reasonable royalty. On January 28, 1946, he announced his decision, with a finding of $950,000 against the Sanitary District for infringement of the patents in the North Side, Calumet (part of original plant), and Des Plaines River sewage treatment works. An appeal was taken by the Sanitary District to the United States Circuit Court of Appeals, which affirmed the decision of Judge Lindley and denied a rehearing.

The United States Supreme Court denied a writ of certiorari and two petitions for a rehearing. The litigation thus ended on May 19, 1947. The case was closed by the payment of the judgment to the various parties concerned, in the sum of $950,000, plus interest, totaling $1,024,047.95. The entire sum collected from all known infringers in the United States totals about $2,497,035.50. This includes $654,987.55 from 115 municipalities, and $818,000 from Milwaukee, in addition to the Sanitary District payment.

Other Litigation on Pollution

The Third Report of this Committee (59), mentioned the pollution case before the United States Supreme Court (No. 11 original) brought by the State of Illinois against the State of Indiana et al to restrain the state and its agencies (four cities and twenty corporations) from polluting Lake Michigan. Stipulations have been drawn up and duly executed by all the respective parties except one. In 1947, the estimated cost of the remedial measures was approximately $11,000,000. In six of the situations (mostly smaller) the necessary remedial work is completed. Because of the difficulty of obtaining engineers, labor, and materials, some of the larger projects have progressed slowly. Quarterly inspections are made by representatives of the states of Illinois and Indiana jointly.

In Maine, a decree was entered in the Supreme Judicial Court in Equity, on December 5, 1947, against the Brown Paper Company, Oxford Paper Company, and International Paper Company, limiting the discharge of waste sulfite liquor and other alleged polluting materials into the Androscoggin River during the period from June 15 to September 15 of each year, and requiring the installation before June 15, 1948, on land, of a lagoon holding 22,000,000 gal to impound the sulfite liquor from the manufacture of at least 400 tons of sulfite pulp by the Livermore Falls Mill of the International Paper Company (normal capacity 500 tons per week) during the period from June 15 to Septembe 15 of each year.

A referee chemist, appointed under the decree, directs the testing, sampling, and analyzing weekly of the Androscoggin River at certain points from May

1 to September 30, to determine, in particular, the dissolved oxygen and hydrogen sulfide. Should the dissolved oxygen appear to be in danger of falling below 4 ppm, the International Paper Company may be ordered to deposit its effluent in the lagoon so that the 4 ppm of dissolved oxygen may be maintained at Turner's Bridge.

Settlement of Damages for Pollution

A manufacturing company in Toledo, Ohio (233), dumped chromic acids and cyanides into the Little Miami River and killed several hundred thousand fish. In settlement of a damage suit, the company agreed to pay the state $18,000 and to furnish 200,000 minnows to restock the stream.

Illinois Legislation on Sanitary Districts

In 1945, the Illinois General Assembly amended the statutes relating to the State Sanitary Water Board and to sanitary districts with a population over 1,000,000. Under this amendment, on July 11, 1946, The Sanitary District of Chicago adopted an ordinance for the control and abatement of pollution of water within its boundaries. The powers relinquished by the State Sanitary Water Board include the examination of all sewer plans and control and abatement of pollution, including sewage, industrial wastes, and other wastes. The Sanitary District in 1948 comprised 462.81 sq miles, including 65 municipalities, with an estimated population of about 4,000,000.

Poliomyelitis

A. L. Sabin (234) believes the poliomyelitis virus originates from either the pharynx or the stools and that oral infection is possible. With the abundance of the virus in the human stools and its presence in sewage, the fly may be suspect. In his opinion, the available evidence is not sufficient to prove that the large epidemics occurring in American cities are related to water supply. He concludes that man is the ultimate reservoir and suggests, for the guidance of physicians, that:

> "Measures taken to prevent flies from breeding in or having access to the sludge beds and other parts of metropolitan sewage disposal plants and creeks containing raw human sewage are reasonable and warranted."

C. E. A. Winslow (235) summarizes the present status of poliomyelitis in a challenging editorial, stating that all the recent evidence tends to confirm the concept that poliomyelitis is exclusively a human infection and that the virus is passed rather directly from an infected person to a noninfected susceptible individual. Evidence that the virus may travel from one person to another by more remote channels is lacking. How long the virus may survive in sewage and sewage polluted water is unknown. After thorough investigation, there is no record of the disease having been spread by a common water supply. There are perhaps four or five instances in which small groups of cases have been associated with a common milk supply, and none in which any other common food has been definitely incriminated as the medium of distribution of an explosive epidemic. Mr. Winslow considers that:

"* * * while cleanliness and sanitation are always highly desirable, there is no reason to believe that improved methods of sewage treatment and disposal, more rigid standards for the purification of water supplies, or the dusting of DDT over a city from aeroplanes will have any measurable effect on the incidence of infantile paralysis."

Respectfully submitted,

JOHN A. CLARK	E. T. KILLAM
WELLINGTON DONALDSON	WILLEM RUDOLFS
LINN H. ENSLOW	M. T. SINGLETON
A. L. FALES	F. M. VEATCH
C. E. KEEFER	LANGDON PEARSE, *Chairman*

Committee on Sewerage and Sewage Treatment, January 22, 1948

APPENDIX. BIBLIOGRAPHY OF REFERENCES

(1) **"A Critical Review of the Literature of 1946 on Sewage and Waste Treatment and Stream Pollution,"** by Committee on Research, Federation of Sewage Works Assns., *Sewage Works Journal*, March, 1947, pp. 207–247. (*a*) pp. 244–283.

(2) **"Activities of the National Council for Stream Improvement, Inc.,"** by H. W. Gehm, *Water & Sewage Works*, July, 1946, p. 289.

(3) **"OK 24 Sanitation Research Projects,"** *Engineering News-Record*, September 25, 1947, p. 10.

(4) **"Public Health Service Sees Need of $7.8 Billion Sanitation Program,"** *Sewage Works Engineering*, August, 1947, pp. 418–421.

(5) **"Contributions of the Sanitary Engineering Program of UNRRA to International Health,"** by F. F. Aldridge, *Public Health Reports*, December 12, 1947, pp. 1729–1739.

(6) **"Report of the Committee on Sewage Disposal,"** Eng. Section, Am. Public Health Assn., Cleveland, Ohio, November 12, 1946.

(7) **"Report of the Committee on Sewage Disposal,"** Eng. Section, Am. Public Health Assn., Atlantic City, N. J., October 6, 1947.

(8) **"Water Pollution Control Act,"** Senate Bill No. 418, 80th Cong., 1st Session, 1947.

(9) Barkley-Taft Bill, original Senate Bill No. 418; Spence Bill, House of Representatives Bill No. 315; Elston Bill, House of Representatives Bill No. 470, 80th Cong., 1st Session, 1947.

(10) **"No Federal Pollution Control,"** *Engineering News-Record*, June 12, 1947, p. 95.

(11) **"Federal and State Legislation for Stream Pollution Control,"** by W. J. Scott, *Sewage Works Journal*, September, 1947, pp. 883–889.

(12) **"Proposed Water Pollution Legislation,"** by M. LeBosquet, Jr., *Water & Sewage Works*, November, 1947, pp. 427–430.

(13) Amending the Act of March 10, 1943, entitled **"An Act to Promote the Conservation of Wild Life, Fish, and Game and for Other Purposes,"** Public Law No. 732, 79th Cong., 1946.

(14) Public Health Law No. 292, 80th Cong., 1st Session, Chapter 407, approved July 31, 1947.

(15) **"Pollution Abatement: Appraisal of Current Regulations,"** by A. Anable and R. P. Kite, *Chemical Engineering Progress*, January, 1948, pp. 3–16.

(16) **"Problems and Progress in Handling Stream Pollution Abatement in Connecticut,"** by W. J. Scott, *Journal*, New England Water Works Assn., March, 1947, pp. 68–73.

(17) **"The Activities of the State Sanitary Water Board in Maine,"** by J. E. Hale, *ibid.*, pp. 64–67.

(18) **"Problems and Progress in Handling Stream-Pollution Abatement in Massachusetts,"** by A. D. Weston, *ibid.*, pp. 80–87.

(19) **"Problems and Progress in Handling Stream-Pollution Abatement in New Hampshire,"** by L. W. Trager, *ibid.*, pp. 68–73.

(20) **"Problems and Progress in Handling Stream-Pollution Abatement in Rhode Island,"** by W. J. Shea; *ibid.*, pp. 77–79.

(21) **"Problems and Progress in Handling Stream-Pollution Abatement in Vermont,"** by E. L. Tracy, *ibid.*, pp. 74–76.

(22) **"Joint Legislative Committee on Interstate Cooperation, New York; Progress Report of the Special Committee on Pollution Abatement,"** Albany, N. Y., February, 1947.

(23) **"Pollution Abatement in New York State,"** by M. M. Cohn, *Water & Sewage Works*, August, 1947, pp. 283–284.

(24) **"Clean Stream Program Produces Results,"** *Engineering News-Record*, June 26, 1947, p. 93.

(25) **"West Virginia Pushes Pollution Control,"** *ibid.*, pp. 91–93.

(26) **"Stream Pollution Control in Indiana,"** by R. B. Wiley, *Journal*, A.W.W.A., September, 1946, pp. 1041–1045.

(27) **"Washington Sewer Planning at Highest Level,"** by E. C. Jensen, *Sewage Works Engineering*, July, 1947, pp. 377 and 392.

(28) **"Oregon Ready to Build and Finance Plants,"** by C. M. Everts, Jr., *ibid.*, p. 376.

(29) **"California Plans $200 Million Treatment Program,"** by E. A. Reinke and J. T. Norgaard, *ibid.*, pp. 373–375

(30) **"National Round-up of Pollution Abatement Progress,"** by M. M. Cohn, *ibid.*, October, 1946, pp. 518–550 and November, 1946, pp. 587–588 and 601.

(31) **"Progress Report on the Allegheny County (Pa.) Sanitary Authority,"** by J. F. Laboon, *Engineering News-Record*, January 1, 1948, p. 6.

(32) **"Allegheny County Sanitary Authority Progress and Plans,"** by J. F. Laboon, *Public Works*, January, 1947, p. 33.

(33) **"Incodel—A Decade of Planned Progress, 1936–1946,"** Interstate Comm. on the Delaware River Basin, Philadelphia, Pa.

(34) **"Fifth Annual Report,"** Interstate Comm. on the Potomac River Basin, Washington, D. C., July 1, 1945, to June 30, 1946.

(35) **"Disposal Plant to Alleviate Boston Harbor Nuisance,"** by K. R. Kennison, *Civil Engineering*, Pt. I, June, 1946, pp. 254–257 and Pt. II, July, 1946, pp. 304–307.

(36) **"Hyperion Plant to Clean up Southern California Beaches,"** *Sewage Works Engineering*, July, 1947, pp. 362–365.

(37) **"Report on Sewage Treatment, Columbus, Ohio,"** by Malcolm Pirnie Engrs., Columbus, Ohio, July 15, 1946 (mimeographed).

(38) **"East Bay Municipal Utility District, Report on Sewage Disposal for Special District No. 1,"** by S. A. Greeley, C. C. Kennedy, and N. T. Veatch, Oakland, Calif., July 1, 1946.

(39) **"East Bay Cities Plan Plants to Cost $23,000,000,"** by D. A. Root, *Sewage Works Engineering*, July, 1947, pp. 366–369.

(40) **"Report of the Joint Board Established to Study and to Prepare Plans and Maps for the Disposal of Sewage in the Merrimack River Valley"** (under Chapter 62, Resolves of 1945), by T. R. Camp, Commonwealth of Massachusetts, Senate No. 550, Boston, Mass. (*a*) pp. 171–173.

(41) **"Report of Chief Engineer on County Sanitation District No. 16 of Los Angeles County,"** by A M Rawn, Los Angeles, Calif., February 16, 1946.

(42) **"Los Angeles County Increases Capacity from 4 to 200 m.g.d.,"** by A M Rawn, *Sewage Works Engineering*, July, 1947, pp. 370–372.

(43) **"Portland's Digested Sludge to Be Discharged Into River,"** by J. W. Cunningham, *ibid.*, pp. 357–358 and 375.

(44) **"Report Upon the Collection, Treatment, and Disposal of Sewage and Industrial Wastes of Orange County, California,"** by A M Rawn, C. G. Hyde, and F. Thomas, June 30, 1947.

(45) **"Santa Clara County (Calif.) Sewage Disposal Survey,"** by C. G. Hyde and G. L. Sullivan, July 31, 1946.

(46) **"Sewage Treatment at Military Installations,"** Rept. of the Subcommittee on Sewage Treatment in Military Installations, Committee on Sanitary Eng., National Research Council, Div. of Medical Science, *Sewage Works Journal*, September, 1946, pp. 791–1030. (*a*) pp. 860–861.

(47) **"Utilization of Sewage Grease,"** by W. Donaldson, *ibid.*, May, 1944, pp. 495–503.

(48) *Fats and Feeds Daily*, Pratt Brothers Co., New York, N. Y.

(49) **"Sewage Treatment at South St. Paul, Minnesota,"** by W. H. Cropsey and K. D. Larson, *Sewage Works Journal*, November, 1947, pp. 1051–1060.

(50) *Proceedings*, 2d Industrial Waste Conference, Purdue Univ., Lafayette, Ind., 1946. (*a*) **"Grease Recovery in the Meat Packing Industry,"** by E. N. Mortenson, pp. 28–29. (*b*) **"Brewery Waste Utilization,"** by B. H. Nissen, pp. 82–89. (*c*) **"Vegetable Wastes, Their Availability and Utilization,"** by H. H. Morris, III, pp. 54–69. (*d*) **"The Composition of Metal-Plating Wastes,"** by W. J. Ehlen, pp. 70–81. (*e*) **"Industrial Waste Disposal,"** by D. A. Cotton, pp. 162–165. (*f*) **"Oil Refinery Waste Recovery,"** by M. J. Morgan, J. W. Randolph,

and R. J. Austin, pp. 144–154. (*g*) **"Problems in Paper Mill Waste Disposal,"** by H. W. Gehm and D. E. Bloodgood, pp. 40–45.

(51) **"Recovery of By-Products from Sewage Sludge Developments at Bradford (England),"** by W. H. Hillier, *Civil Engineering and Public Works Review*, October, 1947, pp. 436–440.

(52) **"Exclusion of Waste Oil from the Baltimore Sewage,"** by C. E. Keefer, *Sewage Works Journal*, May, 1944, pp. 489–490.

(53) **"Treatment and Control Methods for Industrial Wastes,"** by L. F. Oeming, *Water & Sewage Works*, March, 1947, pp. 105–110, especially p. 107.

(54) **"Plant Operating Experience,"** by M. M. Cohn, *Sewage Works Engineering*, Installments XIX to XXXVIII, January to December, 1946, and January to December, 1947.

(55) **"Equipment and Tools for Sewage Works,"** by S. G. Hess, *Water & Sewage Works*, April, 1946, pp. 150–151.

(56) **"Gas Engine Maintenance and Operation,"** by L. S. Kraus, *ibid.*, March, 1947, pp. 81–88.

(57) **"First Progress Report of the Committee of the Sanitary Engineering Division on Sewerage and Sewage Treatment,"** *Proceedings*, ASCE, November, 1942, pp. 1559–1584.

(58) **"Second Progress Report of the Committee of the Sanitary Engineering Division on Sewerage and Sewage Treatment,"** *ibid.*, April, 1944, pp. 457–519.

(59) **"Third Progress Report of the Committee of the Sanitary Engineering Division on Sewerage and Sewage Treatment,"** *ibid.*, June, 1946, pp. 799–855.

(60) **"Report on the Administration, Operation, and Maintenance of the Division of Sewage Disposal for the Year 1946,"** by J. W. Ellms, Dept. of Public Utilities, Cleveland, Ohio, November 1, 1947.

(61) **"Air Diffusion Problems at Cleveland's Southerly Plant,"** by G. E. Flower, *Sewage Works Journal*, May, 1946, pp. 533–534.

(62) **"Annual Report, 1942–1944, Ann Arbor, Michigan, Sewage Treatment Plant,"** by G. P. Witcher, *ibid.*, May, 1946, pp. 552–554.

(63) **"Carbon Tetrachloride Cleans Diffuser Plates,"** by W. W. Dubay, *Sewage Works Engineering*, October, 1946, p. 542.

(64) **"The Use of Digested Sludge and Digester Overflow to Control Bulking Activated Sludge,"** by L. S. Kraus, *Sewage Works Journal*, November, 1945, pp. 1177–1190.

(65) **"Digested Sludge—an Aid to the Activated Sludge Process,"** by L. S. Kraus, *ibid.*, November, 1946, pp. 1099–1112.

(66) **"Some Effects of Anaerobic Digestion Upon Sewage Sludge,"** by A M Rawn and E. J. Candel, *Proceedings*, ASCE (publication pending).

(67) **"Inventory of Sewage Disposal Facilities,"** *Engineering News-Record*, November 14, 1946, pp. 112–113.

(68) Data from **"1940 Census and First, Second, and Third Supplements, Sewage Treatment Works,"** U. S. Public Health Service, Washington, D. C., 1941, 1942, and 1943.

(69) **"Standard Practice in Separate Sludge Digestion: Final Report of the Committee of the Sanitary Engineering Division on Sludge Digestion,"** *Transactions*, ASCE, Vol. 103, 1938, pp. 1662–1731.

(70) **"Multiple-Stage Sewage Sludge Digestion,"** by A M Rawn, A. P. Banta, and R. Pomeroy, *ibid.*, Vol. 104, 1939, pp. 93–132.

(71) **"Evaluation of Methods of Heating Sludge,"** by H. J. Miles, *Sewage Works Journal*, March, 1947, pp. 161–169.

(72) **"A New Method of Heating Sludge Digesters,"** by C. E. Keefer, *Water & Sewage Works*, June, 1946, pp. 236–238.

(73) **"Lessons from a Sludge Tank Explosion,"** by W. J. Wenzel, *Engineering News-Record*, February 21, 1946, pp. 77–79.

(74) **"Design of Sludge Handling Facilities,"** by P. E. Langdon, *Sewage Works Journal*, September, 1944, pp. 899–908.

(75) **"Sludge Preheating Is Featured in New York's Sewage Plant Design,"** *Engineering News-Record*, October 4, 1945, pp. 123–126.

(76) **"Sludge Digestion Temperature Control with Live Steam,"** by A M Rawn, *Water Works & Sewage*, Vol. 90, 1943, Reference and Data Section, pp. 246–248.

(77) **"Digester Heating and Mixing Equipment,"** by V. W. Bacon, *Sewage Works Journal*, May, 1944, pp. 534–539.

(78) **"Sludge Heating and Circulation by Direct Steam Injection,"** by C. P. Gunson, *ibid.*, July, 1947, pp. 661–665.

(79) **"Raw Sludge Steam Heated,"** by E. V. A. Quartly, *Sewage Works Engineering*, August, 1947, pp. 425–426.

(80) **"Scum Digestion and Scum Disposal,"** by R. S. Rankin, *Water & Sewage Works*, October, 1947, p. 372.

(81) **"Digestion Tank Scum,"** by W. E. Gerdel, *Sewage Works Journal*, May, 1946, pp. 543–548.

(82) **"The Scum Problem from the Operator's Standpoint,"** by J. Doman, *Water & Sewage Works*, November, 1946, pp. 451–453.

(83) **"Scum-Free Sludge Digesters,"** *American City*, November, 1945, p. 100.

(84) **"Effect of Copper-Bearing Wastes on Sludge Digestion,"** by H. T. Rudgal, *Sewage Works Journal*, November, 1946, pp. 1130–1137.

(85) **"A Study of the Effect of Nickel on Sludge Digestion,"** by W. J. Wischmeyer and J. T. Chapman, *ibid.*, September, 1947, pp. 790–797.

(86) **"Present Status of Sludge Gas Utilization,"** by R. H. Gould, *ibid.* March, 1947, pp. 170–177.

(87) **"Principles of Power Generation with Sewage-Gas Engines,"** by N. C. Wittwer, *Water & Sewage Works*, April, 1946, Reference and Data Data Section, pp. R259–R265.

(88) **"Digester Gas Explosion at Monroe, Mich.,"** by A. J. Jennings, *Sewage Works Journal*, May, 1946, pp. 529–532.

(89) **"Gas Explosion Damages Digester,"** by H. W. Taylor, *Sewage Works Engineering*, January, 1946, p. 10.

(90) **"Digester Gas Explosion at Traverse City, Mich.,"** by G. Forton, *Sewage Works Journal*, July, 1946, pp. 733–735.

(91) **"Damages Paid for Akron's 1941 Sewer Explosion,"** *Sewage Works Engineering,* September, 1946, p. 465.

(92) **"Gas Fire in Sewer Manhole Traced to Sanitary Fill Operation,"** *Engineering News-Record,* December 25, 1947, p. 51.

(93) **"Concentration of Activated Sludge,"** by D. E. Bloodgood, *Sewage Works Journal,* March, 1947, pp. 202–205.

(94) **"Carbide Lime Slurry for Sludge Conditioning,"** by H. M. Leonhard, *Sewage Works Journal,* September, 1947, pp. 796–800. (*a*) pp. 801–805.

(95) **"Carbide Lime Slurry for Sludge Conditioning,"** *Public Works,* November, 1947, p. 57.

(96) **"Incineration Problems,"** by G. J. Schroepfer, *Sewage Works Journal,* July, 1947, pp. 559–579.

(97) **"Thirteenth Annual Report of the Minneapolis–St. Paul Sanitary District for the Year 1945,"** St. Paul, Minn.

(98) **"Annual Report of the Sewage Disposal Commission of the City of New Britain, Conn., for the Year Ended March 21, 1946,"** *Sewage Works Journal,* May, 1947, pp. 502•503.

(99) **"Annual Report, Buffalo Sewer Authority, 1945–1946,"** Buffalo, N. Y.

(100) **"Report of Operation of the Willow Run Bomber Plant, Detroit, Mich., Sewage Treatment Works for the Years 1943 and 1944,"** *Sewage Works Journal,* November, 1946, pp. 1205–1206.

(101) **"Technical Report on the Operation of the Metropolitan Sewage Treatment Plant, 1945 and 1946, Hartford, Conn.,"** Hartford, Conn.

(102) **"Annual Report, San Diego Sewage Treatment Plant, 1945–1946,"** San Diego, Calif., 1946.

(103) **"Annual Report, San Diego Sewage Treatment Plant, 1946–1947,"** San Diego, Calif., 1947.

(104) **"Annual Report for the Fiscal Year Ending June 30, 1945, of the Richmond-Sunset Sewage Treatment Plant of the City and County of San Francisco, Calif.,"** *Sewage Works Journal,* July, 1947, pp. 665–667.

(105) **"Report of Operation of the District of Columbia Sewage Treatment Plant, 1946–1947,"** Washington, D. C.

(106) **"Summary of Plant Operations, 1945, Greater Winnipeg Sanitary District,"** Winnipeg, Man., Canada.

(107) **"American Sewerage Practice,"** by L. Metcalf and H. P. Eddy, McGraw-Hill Book Co., Inc., New York, N. Y., 3d Ed., 1935, Vol. III, p. 700. (*a*) pp. 143–145.

(108) **"Report of the Bureau of Surveys Comprising the Work at the Sewage Experiment Station at Spring Garden,"** Philadelphia, Pa., 1910.

(109) **"Report on Industrial Wastes from the Stockyards and Packingtown in Chicago,"** by G. M. Wisner and L. Pearse, rept. to the Board of Trustees of The Sanitary Dist. of Chicago, Chicago, Ill., 1914, pp. 129–132.

(110) **"Experience of Chicago, Ill., in the Preparation of Fertilizer,"** by W. A. Dundas and C. P. McLaughlin, in **"Dewatering, Incineration, and Use of Sewage Sludge: A Symposium,"** *Transactions,* ASCE, Vol. 110, 1945, pp. 218–240.

(111) **"8-Year Performance Record of the Plainfield Sludge Drier,"** by J. R. Downes, *Water & Sewage Works*, August, 1946, pp. 323–324.
(112) **"Pilot Plant Results on Vacuator and Auxiliary Equipment,"** by J. J. Wirts and A. J. Fischer, *Sewage Works Engineering*, November, 1946, pp. 584–586.
(113) **"Density Currents in Final Settling Tanks,"** by S. R. Kin, *Water & Sewage Works*, November, 1946, pp. 441–445.
(114) **"High Capacity Trickling Filters,"** by J. A. Montgomery, *ibid.*, Pt. I, January, 1946, pp. 35–41; Pt. II, March, 1946, pp. 119–124; and Pt. III, May, 1946, pp. 198–203.
(115) **"High Rate Filter Performance,"** by F. W. Mohlman, *ibid.*, October, 1947, pp. 372–373. (*a*) p. 373.
(116) **"Sewage Oxidation in Ponds, Performance, Operation, and Design,"** by D. H. Caldwell, *Sewage Works Journal*, May, 1946, pp. 443–458.
(117) **"Emergency Land Disposal of Sewage,"** by W. L. O'Connell, Jr., and H. F. Gray, *ibid.*, July, 1944, pp. 729–746.
(118) **"Sewage Irrigation,"** by J. P. Burden, in **"Manual for Sewage Plant Operators, Texas Water Works and Sewerage Short School,"** Austin, Tex., 1946, p. 83, Chapter VII.
(119) **"Chemical Weed Control,"** by R. F. Goudey, *Journal*, A.W.W.A., February, 1946, pp. 186–202.
(120) **"How to Use DDT for Insect Control,"** by L. F. Warrick and G. F. Bernauer, *Water & Sewage Works*, August, 1946, pp. 329–332.
(121) **"Odor Control and Control of Filter Flies,"** by A. A. Hirsch, *ibid.*, June, 1946, pp. 247–250.
(122) **"Possible Uses of DDT against Insect Pests Encountered in Sewage Treatment,"** by E. J. Hansens, *Sewage Works Journal*, May, 1947, pp. 513–517.
(123) **"Report of Maryland-Delaware Water and Sewage Association Meeting,"** *Water & Sewage Works*, June, 1947, p. 225.
(124) **"Report of Pennsylvania Sewage Works Association Meeting,"** *ibid.*, October, 1947, p. 373.
(125) **"Lack of Chlorine Shuts Los Angeles Beaches,"** *Engineering News-Record*, September 18, 1947, p. 72.
(126) **"Chlorine Manual,"** Chlorine Inst., Inc., New York, N. Y., 1947.
(127) **"The Hypochlorite Process for Treatment of Wool Scouring Wastes and for Recovery of Wool Grease,"** by H. A. Faber, *Sewage Works Journal*, March, 1947, pp. 248–257.
(128) U. S. Patent No. 2,429,868; issued to J. L. Campanella, October 28, 1947.
(129) **"The Treatment of Cyanide Wastes by Chlorination,"** by J. G. Dobson, *Sewage Works Journal*, November, 1947, pp. 1007–1020.
(130) **"Treatment Works for Plating Wastes Containing Toxic Metals and Cyanides,"** by G. E. Barnes, *Water & Sewage Works*, August, 1947, pp. 267–271.
(131) **"Cyanide Removal from Metal Finishing Wastes,"** by F. S. Friel and G. J. Wiest, *ibid.*, March, 1946, pp. 97–98.

(132) **"Report of New England Water Works Association Meeting,"** *Water & Sewage Works*, November, 1947, p. 397.
(133) **"Carbide Wastes Affect Chlorination at Duluth,"** by M. D. Labratovich, *Sewage Works Engineering*, February, 1947, p. 102.
(134) **"Effects of Beet Canning Wastes on Sewage Treatment Plant Operation,"** by E. A. Marshall, *Sewage Works Journal*, March, 1947, pp. 266–269.
(135) **"Annual Report of the Niagara Falls Sewage Treatment Plant,"** by E. J. Smith, *ibid.*, September, 1947, pp. 904–907.
(136) **"How to Solve Activated Sludge Problems,"** by E. J. M. Berg, *Sewage Works Engineering*, March, 1947, pp. 148–150 and 174.
(137) **"Sulfur Bacteria Cause Filter Pooling,"** by C. J. Shepherd, *Sewage Works Journal*, March, 1947, pp. 273–274.
(138) **"Activated Sludge Improved by Dual Application of Chlorine,"** by N. Herda, *Sewage Works Engineering*, March, 1946, pp. 160–161.
(139) **"Sludge Density Control,"** by J. A. Tapleshay, *Water & Sewage Works*, March, 1946, pp. 116–118.
(140) **"Sludge Index Control by Chlorination,"** by J. K. Adams, *ibid.*, June, 1946, pp. 240–242.
(141) **"Annual Report on the San Diego, Calif., Sewage Treatment Plant for the Fiscal Year 1945–1946,"** by E. V. A. Quartly, *Sewage Works Journal*, May, 1947, p. 503 (abstracted).
(142) **"Progress Report on Sulfide Control Research,"** by R. Pomeroy and Fred D. Bowlus, *ibid.*, July, 1946, pp. 597–640.
(143) **"The Effect and Control of Hydrogen Sulfide in Concrete Sewers,"** *Leaflet CBI*, Concrete Information, Conservation Bureau, Portland Cement Assn., Chicago, Ill., September, 1947.
(144) **"Upsewer Choroben Treatment of Los Angeles Sewage,"** by R. F. Brown, *Water & Sewage Works*, May, 1947, pp. 181–186.
(145) **"Chemical Treatment of Los Angeles Sewage to Retard Slimes and Sulfides,"** by A. A. Appel, *Sewage Works Journal*, May, 1947, pp. 494–498.
(146) **"Chlorination Curbs Beach Pollution,"** *Engineering News-Record*, December 11, 1947, pp. 105–107.
(147) **"Pollution Survey of Lake Michigan in Lake County, Illinois,"** by C. W. Klassen, Illinois State Sanitary Water Board and State Dept. of Public Health, Chicago, Ill., September, 1947 (mimeographed).
(148) **"Report of the Joint Board Established for an Investigation of the Sewage Problem in the South Essex Sewerage District,"** *House No. 1250*, Commonwealth of Massachusetts, Boston, Mass., November 28, 1934, pp. 63–65.
(149) **"A Critical Review of the Literature of 1945 on Sewage and Waste Treatment and Stream Pollution,"** by Committee on Research, Federation of Sewage Works Assns., *Sewage Works Journal*, March, 1946, pp. 244–283.
(150) **"Industrial Wastes,"** *Industrial and Engineering Chemistry*, May, 1947, pp. 560–682.

(151) **"Waste Treatment Guide,"** Interstate Comm. on the Potomac River Basin, Washington, D. C., 1947.

(152) *Proceedings*, 1st Industrial Waste Conference, Purdue Univ., Lafayette, Ind., 1944.

(153) **"An Industry Initiates a Waste Control Program,"** by J. E. Cooper, *Sewage Works Journal*, September, 1947, pp. 817–826.

(154) **"Acid Mine Drainage,"** by J. R. Hoffert, *Industrial and Engineering Chemistry*, May, 1947, pp. 642–646.

(155) **"Beet Sugar Industry,"** by B. M. McDill *ibid.*, pp. 657–669.

(156) **"Brass and Copper Industry,"** by W. S. Wise, B. F. Dodge, and H. Bliss, *ibid.*, pp. 632–636.

(157) **"Canning Industry,"** by E. F. Eldridge, *ibid.*, pp. 619–624.

(158) **"Freezing as a Factor in the Stabilization of Corn Cannery Wastes,"** by P. W. Riedesel and W. R. Lawson, *Sewage Works Journal*, September, 1945, pp. 952–965.

(159) **"Citrus Canning Industry,"** by R. R. McNary, *Industrial and Engineering Chemistry*, May, 1947, pp. 625–627.

(160) **"Pollution Control at a Large Chemical Works,"** by J. T. Harlow and T. J. Powers, *ibid.*, pp. 572–577.

(161) **"Coke and Gas Industry,"** by H. A. Gollmar, *ibid.*, pp. 596–601.

(162) **"Gas Works Effluents and Ammonia,"** by Arthur Keyes, Inst. of Gas Engrs., London, 1938.

(163) **"Coal Washery Plants,"** by W. J. Parton, *Industrial and Engineering Chemistry*, May, 1947, pp. 646–652.

(164) **"Corn Starch Process,"** by R. E. Greenfield, G. N. Cornell, and W. D. Hatfield, *ibid.*, pp. 583–588.

(165) **"Dairy Industry,"** by H. A. Trebler and H. G. Harding, *ibid.*, pp. 608–613.

(166) **"Recovery of Fermentation Residues as Feeds,"** by C. S. Boruff, *ibid.*, pp. 602–607.

(167) **"Detection and Measurement of Stream Pollution,"** by M. M. Ellis, *Bulletin No. 22*, Bureau of Fisheries, U. S. Dept. of Commerce, Washington, D. C., 1937, p. 427.

(168) **"Treatment of Cyanide and Acid Plating Wastes,"** by N. Herda, *Sewage Works Journal*, May, 1946, pp. 499–502.

(169) **"Stream Pollution Problems of the Electro-Plating Industry,"** by L. F. Oeming, *ibid.*, July, 1946, pp. 678–685.

(170) **"The Determination of Cyanides,"** by T. R. Haseltine, *Water & Sewage Works*, May, 1947, pp. 187–191.

(171) **"Packing House Industry,"** by F. W. Mohlman, *Industrial and Engineering Chemistry*, May, 1947, pp. 637–641.

(172) **"Industrial Waste Disposal for Petroleum Refineries and Allied Plants,"** by W. B. Hart, *Petroleum Processing*, January, 1946, through September, 1947 (reprint of 20 articles).

(173) **"Pharmaceutical and Biological Plants,"** by E. L. Knoedler and S. H. Babcock, Jr., *Industrial and Engineering Chemistry*, May, 1947, pp. 578–582.

(174) **"The Industrial Waste Problem: III. Recent Developments in Pulp and Paper Waste Treatment Research,"** by H. W. Gehm, *Sewage Works Journal*, September, 1947, pp. 827–829.

(175) **"Characteristics and Methods for Treatment of De-Inking Wastes,"** by W. W. Hodge and P. F. Morgan, *ibid.*, pp. 830–864.

(176) **"Waste Pickle Liquor,"** by R. D. Hoak, *Industrial and Engineering Chemistry*, May, 1947, pp. 614–618.

(177) **"Waste Problems of the Iron and Steel Industries,"** by W. W. Hodge, *ibid.*, November, 1939, pp. 1364–1381.

(178) **"Rubber Industry,"** by G. M. Hebbard, S. T. Powell, and R. E. Rostenback, *ibid.*, May, 1947, pp. 589–595.

(179) **"Tanning Industry,"** by R. Sutherland, *ibid.*, pp. 628–631.

(180) **"Textile Industry,"** by J. C. Geyer, *ibid.*, pp. 653–656.

(181) **"The Purification of Some Textile and Other Factory Wastes,"** by H. Stabler and G. H. Pratt, *Water Supply Paper No. 235*, U. S. G. S., U. S. Govt. Printing Office, Washington, D. C., 1909.

(182) **"A Summary of Experience Gained in the Treatment of the Wastes from the Scouring of Wool,"** by H. R. Crohurst and A. D. Weston, *Engineering and Contracting*, November 10, 1918, pp. 370–376.

(183) **"Recovery of Potash from Wool Scouring Waste,"** by R. S. Weston, *Textile World Journal*, January 12, 1918, p. 255.

(184) **"Modern Sewage Disposal,"** Federation of Sewage Works Assns., New York, N. Y., 1938, pp. 351–371, Chapter XXXII.

(185) **"The Treatment of Textile Wastes,"** by R. S. Weston, *Sewage Works Journal*, July, 1939, pp. 657–674.

(186) **"On Sewage Treatment and Disposal,"** by T. Wardle, J. Heywood, London, 1893, p. 51.

(187) **"Modern Methods of Sewage Purification,"** by C. B. Kershaw, C. Griffin & Co., Ltd., London, 1911, p. 124.

(188) **"Disposal and Purification of Factory Wastes or Manufacturing Sewage,"** by H. W. Clark, 41st Annual Rept., Mass. State Board of Health, Boston, Mass., 1910.

(189) **"Clarification of Wool Scourers' Effluent and Recovery of Wool Fats,"** by A. J. deRaeve, *Industrial and Engineering Chemistry*, August, 1925, p. 837.

(190) U. S. Patent No. 1,543,324; issued to A. J. deRaeve, June 23, 1925.

(191) **"Industrial Waste Interference with Activated Sludge Process,"** by E. L. Nance, *Sewage Works Engineering*, September, 1946, p. 470.

(192) **"Report on the Operation of the Cranston, R. I., Sewage Treatment Works for the Year 1944–1945,"** by W. H. Brown, Jr., *Sewage Works Journal*, March, 1947, p. 278 (abstracted).

(193) **"Effect of Chromate Wastes on the Activated Sludge Process at the Tallmans Island Plant,"** by G. P. Edwards and F. E. Nunsberger, *ibid.*, July, 1947, pp. 598–602.

(194) **"Inhibition of Sludge Digestion by Penicillin Manufacturing Wastes,"** by F. W. Gilcreas, *Sewage Works Engineering*, July, 1946, p. 360.

(195) **"One World or None,"** McGraw-Hill Book Co., Inc., New York, N. Y., 1946.

(196) **"Atomic Energy for Military Purposes,"** by H. D. Smyth, Princeton Univ. Press, Princeton, N. J., 1945.

(197) **"Engineering Features of the Atomic Plants,"** by A. C. Klein, *PR–31878*, Office of the Publication Board, U. S. Dept. of Commerce, Washington, D. C., April 30, 1946 (photostat).

(198) **"Atomic Bomb Engineering,"** by A. C. Klein, *PR–31879*, Office of the Publication Board, U. S. Dept. of Commerce, Washington, D. C., November 20, 1945 (photostat).

(199) **"Water Problems in the Industrial Application of the Utilization of Atomic Energy,"** by I. Perlman, *PB–50939*, Office of the Publication Board, U. S. Dept. of Commerce, Washington, D. C., October 24, 1946 (photostat).

(200) **"Review of Water Monitoring Procedures at Clinton Laboratories,"** by H. M. Parker, *Series C–AVP–52745*, Office of the Publication Board, U. S. Dept. of Commerce, Washington, D. C., June 19, 1946 (photostat).

(201) **"Health Protection in the Production and Use of Atomic Energy,"** by W. F. Bale, *PB–31890*, Office of the Publication Board, U. S. Dept. of Commerce, Washington, D. C., March 27, 1946 (photostat).

(202) **"Sewage Effluents for Industrial Use,"** by N. T. Veatch, *Engineering News-Record*, July 31, 1947, pp. 147–153.

(203) **"Sewage Reclamation,"** by R. F. Goudey, Appendix IV to rept. by H. Conkling to West Basin Water Assn., Los Angeles County, California, July, 1946.

(204) **"Protection of Underground Water from Sewage and Industrial Wastes,"** by A. Pickett, *Sewage Works Journal*, May, 1947, pp. 464–472.

(205) **"Cooling-Water Benefits from Increased River Flows,"** by M. LeBosquet, Jr., *Journal*, New England Water Works Assn., June, 1946, pp. 111–116.

(206) **"Mechanical Equipment of Buildings,"** by L. A. Harding and A. C. Willard, John Wiley & Sons, Inc., New York, N. Y., 1st Ed., 1917, Vol. II, pp. 450–452.

(207) **"The Temperature of the Bottom Deposits of Lake Mendota,"** *Transactions*, Wisconsin Academy of Sciences, Arts and Letters, Madison, Wis., Vol. 23, 1927.

(208) **"Report to the International Waterways Commission,"** Joint Board of Engrs., Washington, D. C., 1926.

(209) **"The Annual Temperature Cycle of Lake Michigan,"** by I. P. Church, *Miscellaneous Reports No. 4*, Inst. of Meteorology, Univ. of Chicago, Chicago, Ill., 1942.

(210) **"The Winter Temperature Cycle of the St. Lawrence Waters,"** by J. G. G. Kerry, *The Engineering Journal*, January, 1946, pp. 26–34.

(211) **"Problems in Disposal of Sludge and Wash Water for Connecticut Water Filtration Plants,"** by F. O. A. Almquist, *Journal*, New England Water Works Assn., 1946, p. 344.

(212) **"Lime and Lime-Soda Sludge Disposal,"** by W. W. Aultman, *Journal*, A.W.W.A., December, 1947, pp. 1211–1214.

(213) **"Brine Disposal from Sodium Zeolite Softeners,"** by P. D. Haney, *ibid.*, pp. 1215–1218.

(214) **"Disposal of Wash Water from Purification Plants,"** by H. R. Hall, *ibid.*, pp. 1219–1223.

(215) **"Water-Carriage Collection of Garbage,"** by H. E. Babbitt, *Bulletin No. 32*, Am. Public Works Assn., Chicago, Ill., November, 1945.

(216) **"Ground Garbage, Its Effect Upon the Sewer System and Sewage Treatment Plant,"** by S. L. Tolman, *Sewage Works Journal*, May, 1947, pp. 441–460. (*a*) pp. 461–463.

(217) **"Effect of Ground Garbage on Sewage Treatment,"** by W. Rudolfs, *Water & Sewage Works*, April, 1946, p. 158.

(218) **"Utilization of Sewage Sludge as Fertilizer,"** *Manual of Practice No. 2*, Federation of Sewage Works Assns., New York, N. Y., 1946.

(219) **"Facts and Fancies about Fertilizer,"** by F. E. Bear, *Plant Food Journal*, April, May, June, 1947.

(220) **"Composts, Manures and Inorganic Fertilizers,"** by W. Thomas, *Gardeners' Chronicle of America*, November, 1947, pp. 298–299 and 308 and December, 1947, pp. 327 and 334–335.

(221) **"The Value of Soil Organic Matter,"** by E. Truog, 5th Annual Conference on Conservation, Nutrition and Health, Ohio State Univ., Columbus, Ohio, *American Fertilizer*, March 22, 1947, p. 26.

(222) **"Trace Elements in Plants and Animals,"** by W. Stiles, Macmillan Co., New York, N. Y., 1946.

(223) **"Fifth Report,"** Royal Comm. on Sewage Disposal, H. M. Stationery Office, London, 1908, Appendix VI, pp. 153–164.

(224) **"Studies on the Digestion of a Sewage Filter Effluent by a Small and Otherwise Unpolluted Stream,"** by R. S. Weston and C. E. Turner, Sanitary Research Laboratory, Mass. Inst. of Tech., Boston, Mass., Vol. X, 1917.

(225) **"Report of the Committee on Sewage Disposal, Engineering Board of Review of The Sanitary District of Chicago,"** Pt. III, Chicago, Ill., February 21, 1925.

(226) **"Dense Algal Growths in Streams Stimulated by Sewage Effluent,"** by G. L. Hall, *Sewage Works Engineering*, February, 1947, p. 94.

(227) **"Fertilization of Lakes by Agricultural and Urban Drainage,"** by C. N. Sawyer, *Journal*, New England Water Works Assn., June, 1947, pp. 109–127.

(228) **"A Study of the Pollution and Natural Purification of the Ohio River: II. Report on Surveys and Laboratory Studies,"** *Bulletin No. 143*, U. S. Public Health Service, Washington, D. C., July, 1924, p. 172.

(229) **"Plankton Productivity of Certain Southeastern Wisconsin Lakes as Related to Fertilization,"** by J. B. Lackey and C. N. Sawyer, *Sewage Works Journal*, May, 1945, p. 575.

(230) **"The Nitrogen Balance of Large Quantities of Water,"** by C. H. Mortimer, *Official Circular*, Public Health Cong. and Exhibition, 1938, British Water Works Assn., Vol. 21, 1939.

(231) **"Activated and Digested Sewage Sludge in Horticulture,"** by A. Howard, Inst. of Sewage Purification, November 20, 1946 (preprint).

(232) **"Loss of Sanitary Sewage through Storm Water Overflows,"** by J. E. McKee, *Journal*, Boston Soc. of Civ. Engrs., April, 1947, pp. 55–92.

(233) **"Manufacturer Pays Ohio for Waste-Killed Fish,"** *Engineering News-Record*, November 21, 1946, p. 15.

(234) **"The Epidemiology of Poliomyelitis,"** by A. L. Sabin, *Journal*, Am. Medical Assn., June 28, 1947, pp. 749–756.

(235) **"Panic or Reason in Dealing with Poliomyelitis,"** by C. E. A. Winslow, *American Journal of Public Health*, August, 1946, pp. 916–918.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

THE SIGNIFICANCE OF PORE PRESSURE IN HYDRAULIC STRUCTURES

Discussion

BY ROSS M. RIEGEL, AND DOUGLAS MCHENRY

ROSS M. RIEGEL,[41] M. ASCE.—The author of this interesting paper choses to rest his case upon deductive reasoning and seems to distrust experimental evidence, even when it appears to support his conclusions to an important degree. To the writer this presentation would be more convincing if the author were to welcome all experimental work, the conclusions of which would tend to support his argument. This would be especially true if the conclusions were reached by different methods of approach. The writer has reference specifically to the work of Karl Terzaghi,[7] Hon. M. ASCE, and of S. Leliavsky Bey,[8] M. ASCE.

There has been a wide divergence of thought among engineers on this point of effective area. As recently as 1945, a book on engineering for dams,[42] after summarizing the existing evidence on the subject, states:

> "The only conclusion that can be drawn from a study of available discussions on uplift is that the value of the area factor 'C' for masonry and rock is uncertain."

It is true that the book subsequently advocates the use of 100% area, but the writers of the book appear to be somewhat doubtful of the Terzaghi experiments, although the latter were published in 1934 and 1936.

In June, 1948, Douglas McHenry, M. ASCE, presented evidence pointing to a boundary porosity of 1.00, which conforms to Professor Terzaghi's con-

NOTE.—This paper by L. F. Harza was published in the December, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1948, by William P. Creager, J. S. Kendrick, Adolf A. Meyer, E. Montford Fucik, A. C. R. Albery, and W. H. R. Nimmo; and September, 1948, *Proceedings*, by John S. Cotton, Serge Leliavsky Bey, A. H. Davison, R. E. Ballester, John S. McNown, and V. T. Boughton.

[41] Head Civ. Engr., Design Dept., TVA, Knoxville, Tenn.

[7] "Simple Tests Determine Hydrostatic Uplift," by Karl Terzaghi, *Engineering News-Record*, June 18, 1936, p. 872.

[8] "Experiments on Effective Uplift Area in Gravity Dams," by S. Leliavsky Bey, in "Uplift Pressure in and Beneath Dams: A Symposium," *Transactions*, ASCE, Vol. 112, 1947, p. 444.

[42] "Engineering for Dams," Vol. II, by Julian Hinds, William P. Creager, and Joel D. Justin, John Wiley & Sons, Inc., New York, N. Y., 1945, p. 267.

clusions.[43] In this case the investigation was made with the new triaxial testing machine of the U. S. Bureau of Reclamation, using high pressures.

To the writer the confirmation offered by the McHenry tests is very convincing so far as laboratory experimentation is concerned, and the author should welcome the support offered to his conclusion, even if the method of approach should be entirely different from his own.

In one other respect the argument of the author might perhaps be strengthened. In so far as the writer knows there has been no failure of a dam which has been traceable to pore pressure in the concrete. (There have been failures in foundations as the result of inadequate resistance to sliding, in which cases uplift was probably an important factor.) This suggests that concrete subject to internal pore pressure still has important properties of strength and is capable of resisting stresses in tension, compression, and shear. The author's analysis and illustrations create the impression that his ideal colums between pores are cylinders. If he were to state clearly that his ideal columns are prismatic in form and are everywhere (except where pores interrupt) in intimate-contact (or continuous) with adjacent prisms, then he could also state that stresses in shear, tension, and compression may be transmitted through these contacts without disturbing the structural integrity of the concrete. The conception of continuity appears to be essential. This idea may be implied in the paper but the writer has failed to find a clear expression of it and can see no reason why it should not be stated. In the writer's view, the reasoning of the author would not be impaired, but would actually be strengthened.

This paper, as interpreted herein, together with the experimental evidence now available, offers considerable support to a design assumption of an effective area of 100%. In so far as this assumption is in excess of that used on so many structures which have functioned satisfactorily for so many years, it is well to recall the effectiveness of drainage (designed or accidental) upon intensities of pore pressure, and to recall also the effects of load transfer from loss of the heat of hydration and other effects[44] in modifying stress conditions established by conventional calculations.

DOUGLAS MCHENRY,[45] M. ASCE.—Questions regarding the effect of pore pressure on the strength and stability of concrete structures seem to present some elusive aspects which make answers by deductive reasoning particularly difficult. The writer has seen a lengthy interchange of correspondence that took place about 1920 to 1930 between two of the most eminent consulting engineers in the United States where both visualized concrete as a series of four-legged tables piled one above the other to form a tower. The stresses in the legs were computed with care, and then a man was placed on the top of each table and instructed to push upward with a certain force against the table above him. The effect on the strength and the stability of the tower was then discussed. These deliberations, however, led to two wholly conflicting views regarding the effect of pore pressure in hydraulic structures. Other discussions

[43] "The Effect of Uplift Pressure Upon the Shearing Strength of Concrete," by Douglas McHenry, *Transactions*, 3d International Cong. on Large Dams, Stockholm, Sweden, 1948 (publication pending).

[44] "Structural Features of Hydraulic Structures," by Ross M. Riegel, in "Design Developments—Structures of the Tennessee Valley Authority: A Symposium," *Transactions*, ASCE, Vol. 111, 1946, p. 1159.

[45] Head, Structural Research Section, Bureau of Reclamation, U. S. Dept. of the Interior, Denver, Colo.

which have done little to clarify the problem have concentrated upon the forces resulting from frictional resistance and upon effects of differences in permeability of cement paste and aggregate.

The writer finds himself in the position of agreeing, for all practical purposes, with the author's conclusion regarding the area acted on by uplift pressure, but disagreeing with the reasoning behind that conclusion and disagreeing with his objections to the experimental method of determining the area. The writer believes that the only method of determining this area is by experiment.

The sense of the author's basic thought may be expressed briefly as follows: The entire solid volume of concrete can be divided into columns of irregular section and length, each terminating in a pore at the top and at the bottom. These pores will be under hydrostatic head differing by their differences in elevation, so that each column will be subjected to buoyancy. Because each column is buoyant, the entire internal structure is buoyant.

This contention appears to be adequately proved by the argument presented in the paper. However, "internal buoyancy" is a new concept in engineering, and the author has apparently assumed rather than investigated its effect on strength and stability. In this discussion, an attempt will be made to carry the reasoning a little further by the presentation of very elementary concepts, with no further apology than that of the author, who states that an elementary treatment seems to be necessary to clarify the subject.

Let one of the author's elementary columns be simulated by an enlarged model. Partly for convenience and partly to follow conventional assumptions, assure that the material has no tensile strength by building the column of blocks or laminae placed one above the other. The laminae fit so well together that if the column is immersed in water the joints will remain dry; but these joints, although transmitting no tension, have a known coefficient of sliding friction. It is desired, first, to determine the weight of the column in air. This may be done by placing the column in a scalepan and weighing it; or the force required to start sliding at any of the joints may be measured. Both methods are tried, and it is found that the unit weight is, say, for convenience, 62.5 lb per cu ft. Now the column is immersed in water and its immersed weight determined by the two methods. (To assure that no phenomena except buoyancy are involved, it may be preferable to immerse the column so that its top surface is flush with the water surface. Deeper immersion will not alter the argument, but will require certain corrections for the direct effects of the water pressure.) The first method (in the scalepan) gives a weight of zero; the sliding method gives 62.5 lb per cu ft. Continuing the experiment, a measurement is made of the force required to separate, by vertical pulls, the two laminae at, say, the midheight of the column. It is found that this requires the same force for the immersed column as for the column in air, although the lifting force for the immersed column reduces to zero as soon as separation occurs. Then the column is reunited, its lower half is fixed in a vertical position, and the force required to start an overturning action of the upper half is measured. The force is the same regardless of whether the column is in air, with no buoyancy, or whether it is in water with full buoyancy. Of course, for the immersed column the overturning force reduces to zero (practically) as

soon as the overturning action has started. Thus, although buoyancy has reduced the scale weight of the column to zero, in all respects which have to do with its strength and stability it behaves in accordance with its true weight of 62.5 lb per cu ft.

If one of the joints in the column is so imperfect that water enters it and acts over practically its entire area it will, of course, be found that sliding and separation at this joint require no force, for they will depend upon the buoyant weight of the material above. This imperfect joint may be looked upon as simulating a pore in the concrete.

Thus, there is a seeming paradox: Concrete dams may be fully buoyant above the rough surface joining the lowest pores, but on any section (plane or irregular), this buoyancy reduces the strength and stability only where the section passes through a pore. Wherever the section passes through solid material, buoyancy may be neglected. Thus, to cause failure at a pore, it is necessary to overcome only the buoyant weight of the column of material above the pore; for failure between the pores, the strength of the material plus the full air weight of the material above must be overcome—not, as the author contends, the buoyant weight of the material. Actually, of course, there is no paradox; the appearance of one arises only if there is read into the expression "internal buoyancy," a meaning which is not there.

If the foregoing reasoning is correct, it may be concluded that uplift pressure (or pore pressure, or buoyancy) should be considered as acting only over that part of a surface which is in direct contact with the pressure fluid. It may be desirable to assume that the entire surface consists of pores, but this assumption is not in the sphere of deductive reasoning, for that question can be settled only by experiment.

Three experimental methods have been used and only one of them has, by any stretch of the imagination, anything to do with internal buoyancy. The most reliable data from the three methods are in reasonable agreement in indicating that pore pressure should be assumed, for practical purposes, to act over 100% of the internal area—in other words, that the boundary porosity is practically unity.

One of the methods employed by K. Terzaghi,[7] Hon. M. ASCE, consists of measuring the relative compressibilities of concrete cylinders subjected to a three-dimensional fluid pressure with and without encasing jackets—that is, with zero pore pressure and with a pore pressure equal to the applied fluid pressure. These tests yielded a value for the boundary porosity of practically 100%. Although the author apparently believes that experiments are unnecessary, and may even be misleading, he puts considerable faith in Professor Terzaghi's tests, and cites them as "complete and conclusive proof" of his own reasoning. However, inasmuch as pore pressures were equal in all parts of these specimens, and therefore buoyancy was not involved, there seems to be no connection between Professor Terzaghi's experiment and the author's reasoning.

The tests[8] by Serge Leliavsky Bey, M. ASCE, in which measurements were made of various combinations of internal pressure and axial load which

produced tensile failure of concrete cylinders, lead to a value of 91%. The author's contention that Mr. Leliavsky misunderstood his own experiment seems unjustified. There is no indication in Mr. Leliavsky's paper that he attributed the failures to the "wedging action" of which the author writes. Inasmuch as a pressure gradient existed through the specimen in these tests, in contrast to the uniform pressure in Professor Terzaghi's tests, something akin to "internal buoyancy" may be involved.

The third method, which has been used in the research laboratories of the Bureau of Reclamation in Denver, Colo., involves determining the effect of pore pressure on the shearing strength of concrete. (A complete record of the tests and the theory of the method is contained elsewhere.[43]) Almost any consistent set of assumptions regarding the nature of the compressive failure of concrete might have been adopted, and would probably have led to about the same numerical result, but it has been assumed that the Mohr-Coulomb theory defines the condition under which compressive failure occurs.

The Coulomb equation is

$$Y = C + X \tan \phi \quad \text{.....................(37)}$$

in which Y is the unit shearing resistance on any given plane; X is the normal stress acting on the plane; C is usually called the unit cohesive strength; and ϕ is the angle of internal friction. Physically, Eq. 37 expresses the hypothesis that the shearing resistance along a plane is equal to a constant plus a linear function of the stress acting normal to the plane. The factors C and ϕ, which are functions of the material, can be evaluated best by triaxial compression tests in which cylinders of concrete encased in rubber jackets are loaded to failure under a circumferential pressure S_3 and an axial pressure S_1.

The fractional part of the area of the failure surface on which pore pressure is acting (called by Professor Terzaghi the boundary porosity n_b) may then be determined by testing additional specimens of the same material with pressure of known intensity acting in the pores. In the simplest testing procedure, the intensity of the pore pressure is made equal to S_3 simply by applying S_3 as a fluid pressure and permitting the fluid to penetrate into the specimen. The pore pressure, being equal in all directions, will not alter the shearing force, but it will reduce the normal stress, X, by the factor $S_3\, n_v$, so that Eq. 37 becomes

$$Y = C + (X - S_3\, n_b) \tan \phi \quad \text{.................(38)}$$

Inasmuch as C and ϕ may be evaluated by Eq. 37, and X and Y may then be determined from the measured quantities S_1 and S_3, Eq. 38 may be solved for n_b.

In the Bureau of Reclamation laboratories, 6-in. by 12-in. concrete cylinders were tested in a triaxial testing machine. The specimens were oven-dried before testing, and nitrogen gas was used as the pressure fluid to assure the rapid development of uniform pore pressure. In all, 337 specimens were tested, representing fifteen different concrete mixes or test ages, with 56 specimens constituting the largest single group. Considerable dispersion was found in the test results, but the method is one in which high precision is very difficult to attain. The linear relationships assumed in the failure criterion appear to

be valid within the limits of experimental error for the range of stresses covered by the tests (S_3 varied from 0 lb per sq in. to 1,800 lb per sq in.; S_1, from 3,270 lb per sq in. to 18,379 lb per sq in.). Values of n_b ranged from 0.78 to 1.18, with an average value of 1.02 with a standard deviation of $\pm$ 0.019, thus indicating that the pressure was effective over practically the entire area of the surface of failure. From the definition of boundary porosity, a value greater than unity is, of course, untenable; but in this method of determining n_b it is to be expected that the experimental values will range about equally above and below the true value. The group of 56 specimens gave a value of n_b equal to 1.07 with a standard deviation of $\pm$ 0.063.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

ELASTIC FOUNDATIONS ANALYZED BY THE METHOD OF REDUNDANT REACTIONS

Discussion

BY JOSEPH GOLD, AND B. LEVINE

JOSEPH GOLD,[23] JUN. ASCE.—The value of this paper lies in its introduction of a much needed approximate analysis for the design of elastic foundations. Unfortunately, the author's introduction of unnecessarily complicated relative deflections may be a deterrent to the wide application of his method. Solely on the basis of Mr. Levinton's paper, the design engineer is unable to decide whether the usual design procedure is adequate or whether a more accurate analysis of the type suggested is warranted. The author also fails to indicate the accuracy obtained in using a specific panel subdivision and neglects to define, adequately, the conditions under which a three-panel, five-panel, or ten-panel solution is sufficient.

Equations similar to those obtained by the author can be derived in a more direct and elementary manner by the method of finite differences. Another advantage of finite differences lies in the existence of various procedures to increase the accuracy of the solution without substantially increasing the calculations required.

Finite differences transform the solution of a differential equation into the solution of a finite number of simultaneous linear algebraic equations. This transformation is accomplished by expressing, approximately, derivatives of all orders of a given function in terms of function values at a discrete number of points. Thus, the first derivative or slope of y at the points x_i and x_{i+1} of Fig. 15 can be approximated by

$$\left.\frac{\Delta y}{\Delta x}\right|_{x_i} = \frac{y_i - y_{i-1}}{\Delta x};\ \left.\frac{\Delta y}{\Delta x}\right|_{x_{1+i}} = \frac{y_{i+1} - y_i}{\Delta x} \text{..............} (32a)$$

The second derivative or curvature of y at x_i is the rate of change of the first

NOTE.—This paper by Zusse Levinton was published in December, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1948, by L. J. Mensch, E. P. Popov, Yi-Mai Yao, Jacob Feld, and Victor R. Bergman.

[23] With Corbett, Tinghir & Co., Inc., New York, N. Y.

derivative and hence is approximated by

$$\left.\frac{\Delta^2 y}{\Delta x^2}\right|_{x_i} = \frac{\left.\frac{\Delta y}{\Delta x}\right|_{i+1} - \left.\frac{\Delta y}{\Delta x}\right|_{i}}{\Delta x} = \frac{y_{i+1} - 2\,y_i + y_{i-1}}{\Delta x^2} \qquad (32b)$$

Eq. 32*b* is the "first-order" central difference operator corresponding to the differential operator $\left.\frac{d^2 y}{dx^2}\right|_{x_i}$. The differential equation governing beam deflections is

$$M = E\,I\,\frac{d^2 y}{dx^2} \qquad (33)$$

in which the moments that produce tension in the upper fibers of a horizontal beam are considered positive, and the deflections downward are positive. The

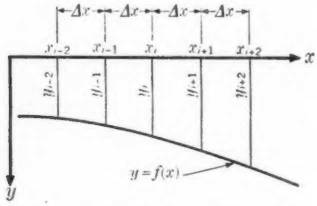


FIG. 15.—DEFLECTION DIAGRAM

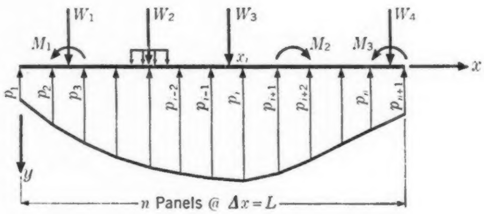


FIG. 16.—*n* PANEL SOLUTION

basic assumption of elastic foundation analysis is manifested by the equation:

$$y = k\,p \qquad (34)$$

in which foundation pressures are positive when they are compressions. Values of *k*, the modulus of the foundation,[24] have been tabulated by the Portland Cement Association, for various soils. For a beam on an elastic foundation Eq. 33 becomes

$$M = k\,E\,I\,\frac{d^2 p}{dx^2} \qquad (35)$$

If the beam (Fig. 16) is subdivided into *n* equal segments (or panels) of length Δx, and if $\frac{d^2 p}{dx^2}$ is replaced by its equivalent finite difference operator

$$\left.\frac{\Delta^2 p}{\Delta x^2}\right|_{x_i} = \frac{p_{i+1} - 2\,p_i + p_{i-1}}{\Delta x^2} \qquad (36)$$

then, at the $n - 1$ intermediate points of subdivision, Eq. 35 yields

$$M_{xi} = \frac{k\,E\,I}{\Delta x^2}\,(p_{i+1} - 2\,p_i + p_{i-1}) \qquad (37)$$

for which condition $i = 2, 3, \cdots, n$ and M_{xi} is the moment at the point x_i of the external loads and of the resulting unknown pressures *p*. It is ascertained that there are $n + 1$ unknown pressures to be determined when the beam is divided

[24] "Concrete Pavement Design," Portland Cement Assn., Chicago, Ill., 1946.

into n panels. The remaining two simultaneous equations required for the determination of the pressures p_i are the equations of equilibrium of vertical forces and of moments. Thus, $n + 1$ simultaneous equations for the $n + 1$ unknown pressures can be easily written.

Applying finite differences to the solution of Example 2 (see Figs. 5 and 17), the numerical value of $\frac{k\,E\,I}{\Delta x^2}$ required is $\frac{0.01 \times 432{,}000 \times 5.33}{12^2} = 160$. Symmetry of the loading leaves only three unknown pressures p_1, p_2, and p_3.

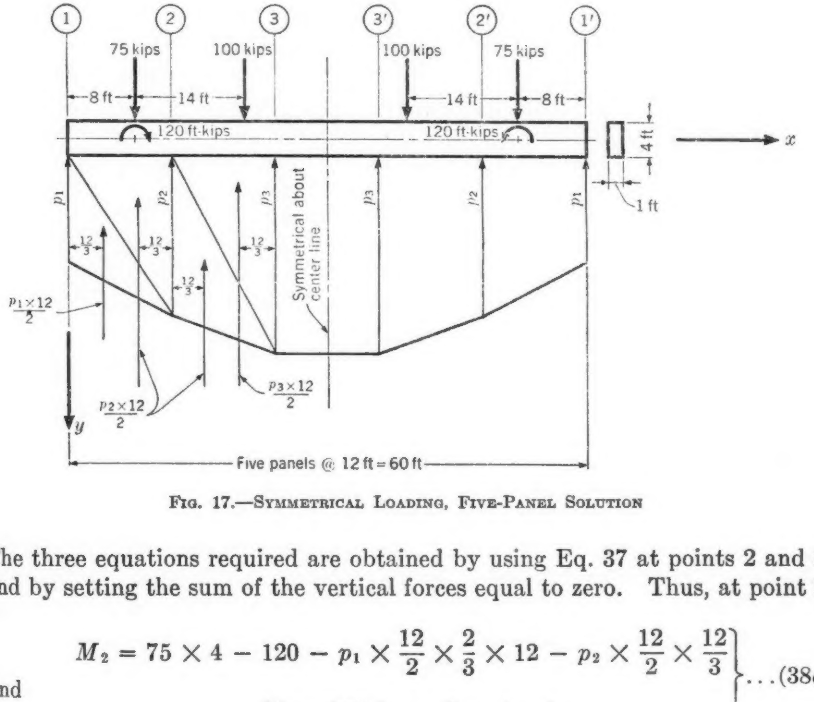


FIG. 17.—SYMMETRICAL LOADING, FIVE-PANEL SOLUTION

The three equations required are obtained by using Eq. 37 at points 2 and 3, and by setting the sum of the vertical forces equal to zero. Thus, at point 2,

$$\left.\begin{aligned} &M_2 = 75 \times 4 - 120 - p_1 \times \frac{12}{2} \times \frac{2}{3} \times 12 - p_2 \times \frac{12}{2} \times \frac{12}{3} \\ \text{and}\quad &M_2 = 160\,(p_3 - 2\,p_2 + p_1) \end{aligned}\right\}\ldots(38a)$$

at point 3,

$$\left.\begin{aligned} M_3 = {} & 75 \times 16 + 100 \times 2 - 120 - p_3 \times \frac{12}{2} \times \frac{12}{3} - p_2 \times \frac{12}{2} \times \frac{2}{3} \times 12 \\ & - p_2 \times \frac{12}{2}\left(12 + \frac{12}{3}\right) - p_1 \times \frac{12}{2}\left(12 + \frac{2}{3}\,12\right) \\ \text{and}\quad & M_3 = 160\,(p_3 - 2\,p_3 + p_2) = 160\,(p_2 - p_3) \end{aligned}\right\}\ldots(38b)$$

and, by equilibrium of vertical forces,

$$75 + 100 + 100 + 75 = 2\left(\frac{p_1 + p_2}{2} \times 12 + \frac{p_2 + p_3}{2} \times 12 + p_3 \times 6\right).(38c)$$

After simplification, this system becomes

$$\left.\begin{aligned} 208\,p_1 - 296\,p_2 + 160\,p_3 &= 180 \\ 120\,p_1 + 304\,p_2 - 136\,p_3 &= 1{,}280 \\ 12\,p_1 + 24\,p_2 + 24\,p_3 &= 350 \end{aligned}\right\} \qquad (39)$$

and its roots—$p_1 = 3.807$, $p_2 = 5.791$, and $p_3 = 6.890$—agree fairly closely with the author's values—$p_1 = 3.930$, $p_2 = 5.820$, and $p_3 = 6.800$—all in kips per sq ft. The positive values of p_1, p_2, and p_3 indicate that the pressures are all compressions, and that the deflections of the beam are all downward. Knowing the pressures, the moments at points 2 and 3, $M_2 = -142$ ft-kips and $M_3 = -176$ ft-kips, are obtained by substitution in Eqs. 38*a* and 38*b*.

The author states in his conclusions, "The remaining work is simply routine solution of simultaneous equations, with various degrees of accuracy, as the case may require." In order to establish the factors determining the accuracy of the solution of a specific problem, a few pertinent remarks are in order. The conventional method for the design of concrete foundations on soil or piles is based on the premise that these foundations are infinitely rigid and do not settle unequally under load. In many cases this conventional analysis yields results that are within the accuracy of the determination of the physical properties and loadings of the structure, and a more refined analysis is not justified. A dimensionless parameter, λ, governing problems of this type is defined by the equation:[25,26]

$$\lambda = \sqrt[4]{\frac{1}{4\,k\,E\,I}}\,L \qquad (40)$$

in which L denotes the length of the beam. The value $\lambda = 0$ corresponds to the conventional assumptions of infinite rigidity. Large values of λ are possible as a result of flexible beams or stiff foundation supports (small k).

To estimate the accuracy of the finite difference method, the two-panel, three-panel, four-panel, and five-panel solutions are compared with the theoretical solution[25] for the beam with a concentrated load at its midpoint. The conclusions reached are also valid for the author's method of redundant reactions in the light of the close agreement between the two methods in the solution of Example 2 and other problems not included in the discussion. This discussion is limited to values of λ between 0 and 6, since a great variety of concrete foundations falls within this range. To maintain the desired dimensionless characteristic, ratios of the maximum pressure (p_{cl}) and maximum moment (M_{cl}) to the conventional values obtained with $I = \infty$, are plotted in Fig. 18. Referring to the ordinate captions of Fig. 18, the conventional values $(M_c)_{cl}$ and $(p_c)_{cl}$ (that is, the maximum center-line values) are, respectively, $\frac{W}{L}$ and $\frac{W\,L}{8}$. Examination of these curves clearly shows that for small values of λ the two-panel solution ($n = 2$) yields results that are close to the theoretical values. For larger values of λ, a greater number of panels is re-

[25] "Strength of Materials," by S. Timoshenko, D. Van Nostrand Co., Inc., New York, N. Y., 1944 Pt. II, pp. 1–24.

[26] "A Symmetrically Loaded Base Slab on an Elastic Foundation," by S. U. Benscoter, *Transactions*, ASCE, Vol. 109, 1944, pp. 763–776.

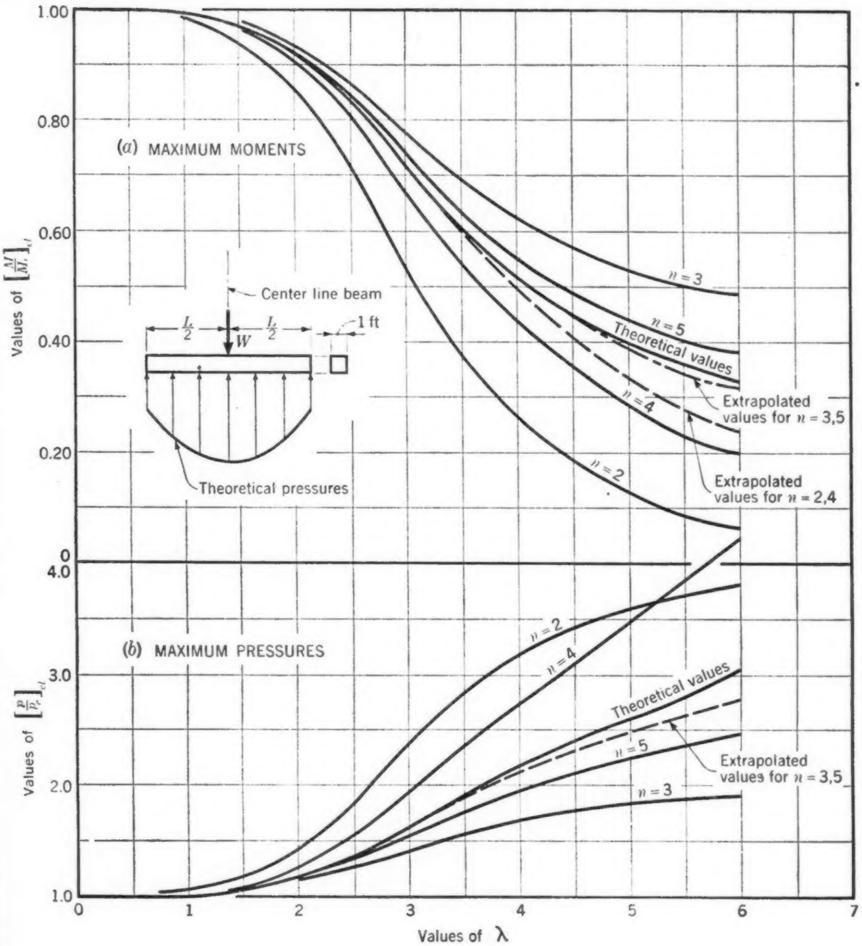


FIG. 18.—COMPARISON OF MAXIMUM PRESSURES AND MOMENTS

TABLE 8.—RECOMMENDED NUMBER OF PANELS FOR AN AVERAGE ACCURACY OF $\pm 10\%$ IN THE DETERMINATIONS OF PRESSURES, SHEARS, AND MOMENTS

Recommended:	RANGE OF λ-VALUES					
	0 to 1	1 to 1.5	1.5 to 2	2 to 3	3 to 4.5	4.5 to 5
Number of panels	Conventional[a]	2	3	4	5	6

[a] Conventional method is sufficient.

quired to approximate the deflected shape and yield reasonably accurate results. As was to be expected, in general, the larger the number of panels, the greater will be the accuracy. As a result of the experience gained in this simple problem, the recommendations contained in Table 8 are made for the usual design problems.

A more refined derivation[27] of the various finite difference operators shows that in Eq. 36, only the first term of an infinite series for the finite difference operator corresponding to $\frac{d^2p}{dx^2}$ appears. Using two terms of this infinite series results in a more accurate expression for $\frac{d^2p}{dx^2}$ depending on five values of p_i in comparison to the three values appearing in Eq. 36. Using this "second-order" central difference operator, Eq. 37 becomes

$$M_{x_i} = \frac{k\,E\,I}{\Delta x^2}\left(-\frac{1}{12}p_{i+2} + \frac{4}{3}p_{i+1} - \frac{5}{2}p_i + \frac{4}{3}p_{i-1} - \frac{1}{12}p_{i-2}\right) \ldots (41)$$

for which condition $i = 2, 3, \cdots, n$. Applying the second-order operators to Example 2 leaves Eq. 38*c*, representing vertical force equilibrium and the first part of Eqs. 38*a* and 38*b*, representing the moments of the external loads and unknown pressures p, unchanged. The only changes resulting are at point 2:

$$M_2 = 160\left(-\frac{1}{12}p_3 + \frac{4}{3}p_3 - \frac{5}{2}p_2 + \frac{4}{3}p_1\right) \ldots\ldots\ldots (42a)$$

and, at point 3:

$$M_3 = 160\left(-\frac{1}{12}p_2 + \frac{4}{3}p_3 - \frac{5}{2}p_3 + \frac{4}{3}p_2 - \frac{1}{12}p_1\right) \ldots\ldots (42b)$$

Simplification yields the system:

$$\left.\begin{array}{r} 261.33\,p_1 - 376\,p_2 + 200\,p_3 \quad = \quad 180 \\ 106.67\,p_1 + 344\,p_2 - 162.67\,p_3 = 1{,}280 \\ 12\,p_1 + \;\; 24\,p_2 + \qquad 24\,p_3 = \quad 350 \end{array}\right\} \ldots\ldots\ldots\ldots (43)$$

and its roots in kips per square feet—$p_1 = 3.774$, $p_2 = 5.808$, and $p_3 = 6.888$ (which are within 1% of the previously computed values)—are more accurate. It is interesting to note that the value of λ for this beam is $\sqrt[4]{\frac{1}{4 \times 0.01 \times 432{,}000 \times 5.33}} \times 60 = 3.44$, and that the five-panel solution which was used is the one recommended in Table 8. As a result of the increased accuracy inherent in the use of second-order operators, the number of panels required to attain a certain accuracy is smaller than that necessitated by the use of first-order operators. L. Collatz[28] and M. G. Salvadori,[29] Assoc. M. ASCE, have shown the advantages of using the more accurate difference operators in the solutions of various linear boundary value problems.

[27] "Central Difference Formulae," by W. F. Sheppard, *Proceedings*, London Mathematical Soc., Vol. 31, 1899, p. 460.

[28] "Zusammendfassender Bericht über die genäherte Berechnung from Eigenverte," by L. Collatz, *Zeitschrift*, Angewandte Mathematik und Mechanik, Vol. 19, 1939, pp. 224–249 and pp. 297–318.

[29] "The Numerical Evaluation of Buckling Loads by Finite Differences," by M. G. Salvadori, paper presented at the Annual Meeting of the ASCE, New York, N. Y., January, 1948.

The error introduced by using the various ordered central difference operators in place of the differential operators has been evaluated[30,31] and an extrapolation procedure has been established and used successfully elsewhere.[29,30,31,32] Generally, at least three approximations are calculated (for example, $n = 2$, 3, 4), and the extrapolation formulas developed are applied to those approximations forming a monotonic sequence. The dotted curves shown in Fig. 18 represent the extrapolated values for the odd and even approximations. The great increase in accuracy inherent in the extrapolation procedure, as compared with that suggested by Mr. Levinton, is obvious upon examination of these curves.

B. LEVINE,[33] Assoc. M. ASCE.—In applying the author's method to a number of actual design problems, the writer has observed its simplicity and time saving features as compared with exact methods of analysis using differential equations. The following data will enable a designer to establish, quickly, the conditions under which it is advantageous to use the author's method.

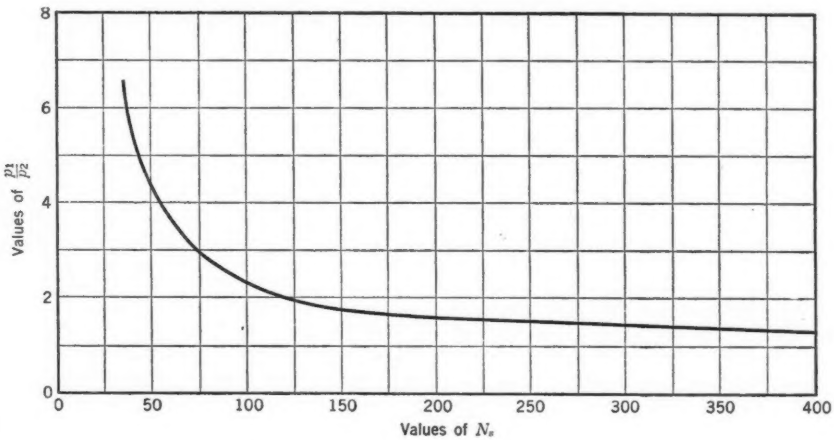


FIG. 19.—RELATION BETWEEN N_s AND $\frac{p_1}{p_2}$ FOR SYMMETRICAL LOADING; THREE-PANEL SOLUTION

Table 9 indicates a foundation slab loaded symmetrically and gives the author's solution for five different types of soils. The soils vary from a gravel-sand mixture having a modulus of subgrade reaction,[24] K, of 500 lb per sq in. per in. to a silt having a subgrade reaction modulus of 100 lb per sq in. per in.

[30] "The Approximate Arithmetical Solution by Finite Differences of Physical Problems Involving Differential Equations," by L. F. Richardson, *Philosophical Transactions*, Royal Soc. of London, Vol. 210, 1911, pp. 307–357.

[31] "On the Evaluation of Characteristic Values by Finite Differences and Richardson's Extrapolations," by M. G. Salvadori, paper to be presented at the 7th International Cong. of Applied Mechanics, London, England, September, 1948.

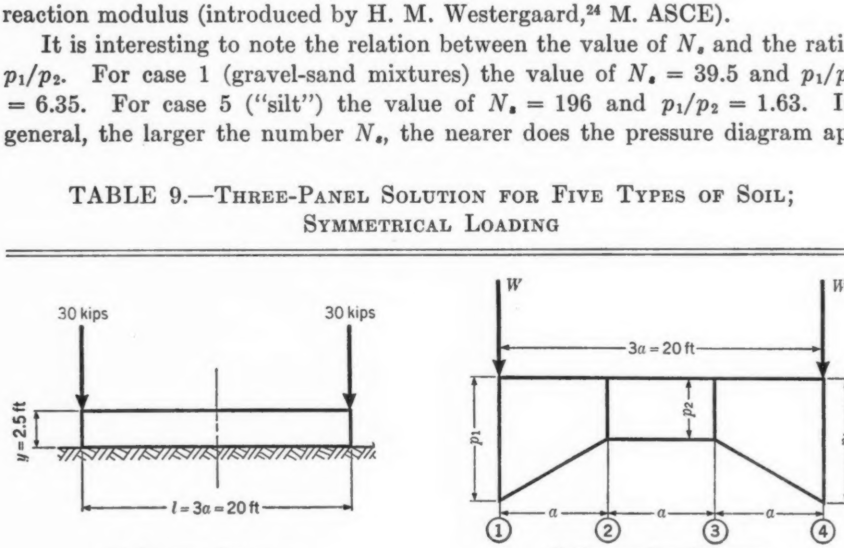
[32] "Buckling Loads for Built-In Rectangular Plates by Finite Differences," by J. Gold, thesis presented to Columbia University at New York, N. Y., in June, 1947, in partial fulfilment of the requirements for the degree of Master of Science in Civil Engineering.

[33] Designing Engr., Knappen Tippetts Abbett Eng. Co., New York, N. Y.

The corresponding values of k, defined by the author as "modulus of foundation material" are obtained by dividing the number $0.579 = \frac{1{,}000}{1{,}728}$ by the subgrade reaction modulus (introduced by H. M. Westergaard,[24] M. ASCE).

It is interesting to note the relation between the value of N_s and the ratio p_1/p_2. For case 1 (gravel-sand mixtures) the value of $N_s = 39.5$ and $p_1/p_2 = 6.35$. For case 5 ("silt") the value of $N_s = 196$ and $p_1/p_2 = 1.63$. In general, the larger the number N_s, the nearer does the pressure diagram approach the condition of uniform soil pressure under the entire length of the foundation beam.

TABLE 9.—THREE-PANEL SOLUTION FOR FIVE TYPES OF SOIL; SYMMETRICAL LOADING



(a) LOADING DIAGRAM

(b) TYPICAL SOIL PRESSURE DIAGRAM

Symbol	Definition	Quantity
W	Applied loads, in kips	60
a	Segment of span length, in feet	6.67
E	Modulus of Elasticity—	
	Pounds per square inch	3,000,000
	Kips per square foot	432,000
I	Moment of inertia, $\frac{1 \times 2.5^3}{12}$ ft^4	1.30
b	Breadth of slab (unity)	1
p	Conventional soil pressure, W/l, in kips per square foot	3
l	Span length, 3 a, in feet	20
M_A	Conventional moment at center line of beam (point A) in foot-kips	150
M'_A	Moment at center line of beam (point A) as computed by three-panel solution, in foot-kips	...
K	Modulus of subgrade reaction	...
k	Modulus of foundation material	...
N_s	Substitution factor defined by Eq. 8	...
Y	Depth of foundation beam, in feet	2.5

Case No.	Type of soil	K	k	N_s	p_1	p_2	p_1/p_2	M'_A	M'_A/M_A
1	Gravel-sand mixtures	500	0.00116	39.5	6.84	1.08	6.35	97	0.64
2	Gravel-sand-clay mixtures	400	0.00145	49.4	6.28	1.36	4.6	104	0.69
3	Sand-clay mixtures	300	0.00193	65.6	5.64	1.68	3.35	112	0.75
4	Fine sand	200	0.00290	98.6	4.9	2.05	2.39	124	0.83
5	Silt	100	0.00579	196.0	4.04	2.48	1.63	136	0.91

A second item of interest to the designer is the value of p_1 as compared with the conventional soil pressure p. For case 1, $p_1 = 6.84$ kips per sq ft as com-

pared with $p = 3$ kips per sq ft. In general, the smaller the number N_s, the greater will be the soil pressure under the ends of the foundation beam as compared with the soil pressure under the middle.

A third item of interest is the reduction in maximum moment at the center of the foundation beam obtained by using the author's solution as compared with the moment obtained by using the conventional method. For case 1 the value of M_A is 64% of the value of M. For cases 1 and 2, this reduction in moment will justify using a foundation beam 2 ft deep, instead of 2.5 ft deep as obtained by conventional analysis.

The curve in Fig. 19 indicates the relation between the number N_s, by Eq. 8, and the ratio p_1/p_2. This curve is obtained as follows: For symmetrical loading the author gives Eq. 7*a*. For the loading shown in the diagrams of Table 9, the term y_{b2}, Eq. 7*a*, becomes zero and that equation may be written:

$$(19 - N_s)\, p_1 + (91 + N_s)\, p_2 = 0 \qquad (44a)$$

or

$$\frac{p_1}{p_2} = \frac{N_s + 91}{N_s - 19} \qquad (44b)$$

The value of N_s may be computed for any particular design problem by substituting the proper values in Eq. 8. Then, by using Fig. 19, the corresponding value of p_1/p_2 may be obtained from the curve.

A study of Fig. 19 indicates that in general a value of N_s of 200 or more corresponds to a foundation condition in which the beam is relatively stiff as compared to the soil. For this condition the conventional design method may be used, since the soil pressure diagram and the value of maximum moment are practically the same as those obtained by the author's solutions. However, for values of N_s less than 200, the three-panel solution enables the designer to visualize clearly (*a*) the distribution and intensities of soil pressures under the foundation beam, and (*b*) the maximum value of moment at the center of the foundation beam and the possibility of reducing the depth of the beam as compared with the depth required by conventional analysis.

The numerical work involved in obtaining the simple beam deflections resulting from loads and moments is quite appreciable; and this work could be reduced considerably by the use of influence lines.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

PANAMA CANAL—THE SEA-LEVEL PROJECT A SYMPOSIUM

Discussion

BY E. MONTFORD FUCIK, CHARLES M. ROMANOWITZ, AND RAPHAEL G. KAZMANN

E. MONTFORD FUCIK,[98] Assoc. M. ASCE.—Messrs. Binger and Thompson have very properly emphasized that the major problems in arriving at proper slopes for the sea-level canal concern two materials—the Cucaracha formation, a rock, and the Atlantic muck, a soil. The authors state that the slope curves developed for use in setting slopes for estimating quantities were based on minimum strengths for the materials, and that further study for detailed design might result in steeper slopes, and reduced excavation quantities. The writer heartily agrees with this statement, especially in regard to slopes for cuts in the Cucaracha formation.

Slopes in the Cucaracha Formation.—Fig. 64 shows the curve of slope versus depth of cut which was used for setting slopes in the Cucaracha formation, and Fig. 71 shows typical slopes for a part of the proposed canal in which the Cucaracha formation is encountered. Referring to section A–A in Fig. 71, a slope of 1 on 9.4 is proposed. The writer believes that it is entirely possible that this slope, and others set by the use of the curve shown in Fig. 64, are too conservative, and that a material steepening of the slopes could be effected without any great risk of incurring catastrophic failures of the banks. The authors state that a stable bank of a cut about 200 ft deep in the Cucaracha formation having a slope of 1 on 2.7 was analyzed to determine the over-all strength of the Cucaracha formation. Seepage forces were considered to act on part of the height of the cut, and the over-all strength of the Cucaracha formation in this bank was obtained by considering a safety factor of unity, which assumes that the bank is on the verge of failure. The cohesive strength obtained in this manner was found to be 16 lb per sq in., with an assumed angle of internal friction of 10°.

NOTE.—This Symposium was published in April, 1948, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: June, 1948, by Hans Kramer, and Philip G. Nichols; and September, 1948, by George B. Pillsbury, Kenneth S. M. Davidson, Joel D. Justin, W. H. McAlpine, W. E. R. Covell, William Herbert Hobbs, Hibbert Hill, Kenneth C. Reynolds, Gregory P. Tschebotarioff, Charles W. Dohn, and Donald F. Horton.

[98] Associate, Harza Eng. Co., Chicago, Ill.

In arriving at the slope versus depth of cut curve used in setting slopes for the estimate, a condition of sudden drawdown over the entire height of cut was assumed, and a factor of safety of 1.3 was introduced. It can be seen that this resulted in slopes as flat as 1 on 9.4 for the cut illustrated in section A–A in Fig. 71. It is the opinion of the writer that the slope versus depth of cut curve finally arrived at is unduly conservative for design purposes, when the over-all problem of the sea-level canal is considered, and that the magnitude of the excavation quantities involved is such that the use of a lesser over-all factor of safety would be entirely justified in the final design analysis.

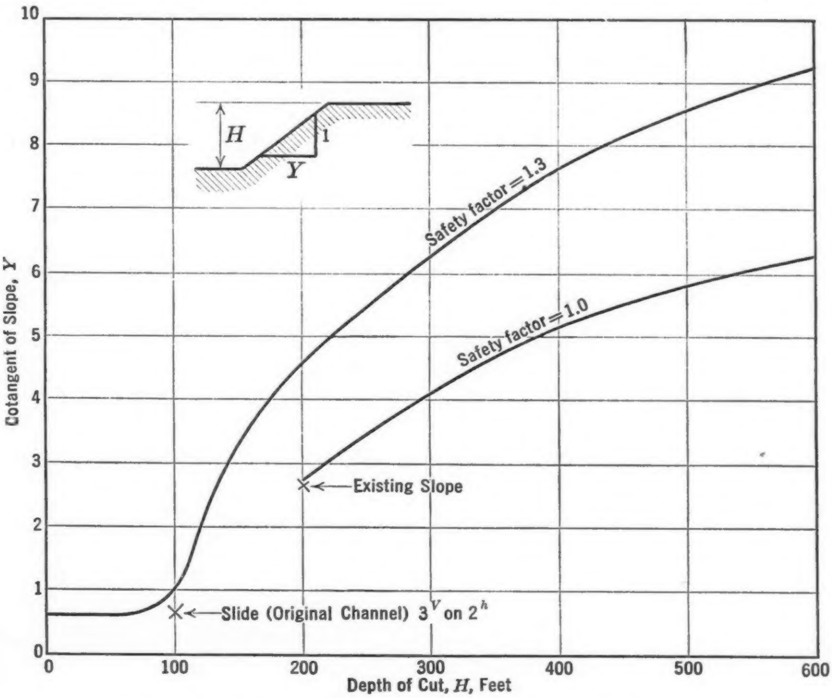


FIG. 110.—EXCAVATION SLOPE CURVE FOR CUCARACHA FORMATION

The strength value for the Cucaracha formation was derived by a careful study of a stable bank of considerable height, after it had been determined by core borings that the material in the bank was typical of the Cucaracha formation. The condition of sudden drawdown was applied to this bank only below a layer having good drainage. This assumption provides a hidden factor of safety in the final slope curve, which was based on sudden drawdown throughout the full depth of the cut. The factor of safety of the stable bank was assumed to be unity, which is the most conservative assumption possible, because the bank has been standing for 30 years and must have a safety factor of at least unity. It is entirely possible that the actual safety factor is as high as 1.2, because no evidence of incipient failure was found in the bank. A factor of

safety of 1.3 was then added to the foregoing unknown safety factor when making the final slope curve in estimating.

In an excavation project as large as the proposed sea-level canal there is considerable question as to whether it is more economical to design every slope so that it will be absolutely safe against failure, or whether the slopes should be set close to the steepest possible slope, by using a low safety factor. If the latter scheme is employed, some slides might possibly occur, but they probably will involve less additional excavation than if all the slopes were originally flattened so that there would be an ample safety factor against all conditions.

Using the Cucaracha formation strength values on which the slope curve in Fig. 64 is based, but using a factor of safety of 1.0 instead of 1.3, the writer has worked out the curve shown in Fig. 110. The slope curve for a factor of safety of 1.3 is also plotted, as are the points showing the stable bank from which the strength constants used in these studies were derived, and the slope of 3 on 2 for a 100-ft bank, which failed during the original construction period.

Fig. 71 shows, in plan, the section of the proposed sea-level canal which cuts through the Cucaracha formation. This section is about 17,000 ft long. If an average depth of cut of 300 ft is assumed for this length, and if only one bank is taken to be in the Cucaracha, as seems to be indicated in Fig. 71, the saving in excavation quantities by using the slope curve for a safety factor of 1.0 (Fig. 110), rather than that for a safety factor of 1.3, is about 65,000,000 cu yd. This estimate indicates the magnitude of quantities involved, and is large enough to merit serious consideration as it represents about 15% of the total volume of the excavation in this section of the canal.

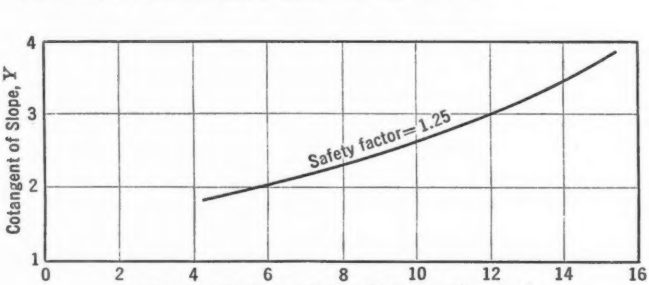


FIG. 111.—SLOPE CURVE FOR ATLANTIC MUCK; FOR CUTS DEEPER THAN 30 FT NOT UNWATERED

It should be pointed out again that the safety factor of 1.0 does not necessarily represent a condition of incipient failure for the slopes, but merely indicates that these slopes have the same safety factor as an existing slope 200 ft high which has been standing for 30 years without any signs of distress. In all probability, slopes designed with the same relative stability as this existing bank would suffer from only minor slides, if at all.

Slopes in Atlantic Muck.—The Atlantic muck was thoroughly tested during the studies for the Third Locks Project, from 1940 to 1942, and slopes designed from these tests have remained stable after excavation, as stated by the authors. The most interesting and unusual condition in connection with

slopes in this material is the great influence which the height of bank above the water level has on the stability for excavations made below sea level and never unwatered. This resulted in the adoption of a uniform slope for all excavation through the Atlantic muck for the proposed sea-level canal. The simplification in dredging a uniform section, rather than a varying section, apparently justifies the removal of some additional yardage.

Fig. 111 shows how much the height of bank above the water affects the slope required for cuts in the Atlantic muck greater than 30 ft and having a safety factor of 1.25. This curve was developed at the Canal Zone during the Third Locks Project.

Conclusions.—The writer's remarks concerning the apparent conservatism of the slope curve for the Cucaracha formation have not been made as a criticism of using Fig. 64 to arrive at estimating quantities for the excavation, because the report of the Governor should present an over-all estimate which is on the high side, rather than on the low side, of the probable cost. However, when, as, and if the sea-level canal becomes an actuality, the writer feels that careful and thorough study should be given to the possibility of using steeper slopes in the Cucaracha formation than are presented in the paper of Messrs. Binger and Thompson.

If the Congress determines to proceed with the construction of a sea-level canal, there will exist an unparalleled opportunity for the advancement of engineering knowledge concerning excavation slopes. The unusual depths of cuts which will be encountered, plus the variety of material to be found—from the softest ocean deposit to extremely hard basalt rock—will yield information which should permit great advancements in the art of excavation slope design.

CHARLES M. ROMANOWITZ,[99] ASSOC. M. ASCE.—Several types of equipment are proposed for the excavation of the sea-level project of the Panama Canal in the paper on "Construction Planning and Methods." The bucket-ladder dredge mentioned therein was the subject of an extensive engineering study carried on by the Yuba Manufacturing Company of San Francisco, Calif.

This company has for many years specialized in bucket-ladder dredge equipment for the mining of placer gold, platinum, and tin ore (cassiterite). A dredge of this type is recommended for the proposed excavation of the rock when suitably broken up by blasting. An experimental blasting project mentioned in the Symposium has proved that rock can be broken up satisfactorily for bucket-ladder dredge excavation.

Two dredges are proposed, each having buckets of 54-cu-ft capacity, one digging to a depth of 90 ft below water level and the other digging to a depth of 145 ft below water level. As designed, these dredges embody the experience of the harbor type, bucket-ladder dredge, and the placer mining type, bucket-ladder dredge, with the units being modernized with proved designs.

A record of the performance of bucket-ladder dredges in the present Panama Canal construction is given by one of the authors of the paper on "Construction

[99] Director of Sales, Yuba Mfg. Co., San Francisco, Calif.

Planning and Methods."[100] Seven bucket-ladder dredges, taken over from the French operations, were found to have inadequate power and bucket capacity (17 cu ft). Although their production was not as great as desired, their performance led to ordering a larger and more powerful dredge, the *Corozal,* which started operation on April 11, 1912. Besides having buckets of 54-cu-ft capacity for soft materials and 34-cu-ft capacity for digging rock, this apparatus could dredge to a depth of 50 ft, dumping the spoil either into hoppers on the dredge or into barges alongside. In soft to medium hard materials its maximum monthly yardage was 176,574 cu yd; in 100% rock, 173,267 cu yd. Considerable trouble was caused by breakages, although when digging the *Corozal* gave a good account of itself.

Bucket-ladder dredges are used extensively for placer mining and require a large space on board for treating the materials dug. This has been the main factor in limiting the bucket size of the largest placer mining dredges to 18 cu ft each, although such dredges have not reached a maximum digging depth. In California there are dredges digging the deepest gold placer ground on record —one digging to 112 ft below water level, and the other digging to 124 ft below water level with banks above water up to 50 ft high, making a total depth of 174 ft below ground level. In the same area some of the property is to be equipped with dredges digging to greater depths below water level than those proposed for the Panama Canal sea-level project.

In Malaya, according to official records, there were in June, 1940, eight dredges digging to depths from 110 ft to 130 ft below water level, with five of these digging to the greater depth. It has been stated by responsible mining men in Malaya, that dredges digging to depths of at least 150 ft below water level will be considered in the future.

The bucket-ladder dredge developed rapidly because values found in placer gravels are low. Every factor that increases the efficiency of the dredge must be given serious consideration. Most of the innovations and improved designs have come directly either from the operators to the designing engineers, or from the engineers who have developed them as a result of having direct contact with the operations. This close association has contributed greatly to the rapid development of the California type dredge.

Credit must also be given to the manganese steel foundries serving the placer mining industry. They have contributed much to the development of the buckets, an important unit on the dredge. Although the largest placer mining dredge has buckets of 18-cu-ft capacity, which is only one third of the capacity of the largest buckets used for harbor work, there is a substantial difference between the two types of buckets. The placer mining bucket is an efficient cutting and digging tool, equally efficient in dumping, and capable if necessary of dislodging large boulders and digging hard bedrock. It has great strength and resistance to wear which makes it a unit of long service. It also has pins of large diameter as compared with those used on harbor dredge buckets, and a bolted lip that is quickly replaceable.

[100] "Dredging with Ladder Dredges," by Forrest L. Dye, *Memorandum No. 19,* Isthmian Canal Studies, The Panama Canal, Diablo Heights, Canal Zone, July 8, 1946.

A placer dredge is expected to show an average daily running time of at least 21 hours for a period of several years. There are cases, under extreme conditions, where running time as low as 18 hours daily is considered satisfactory. The yardage dug by placer mining dredges naturally depends on the conditions and the formations encountered. A dredge having buckets of 12-cu-ft capacity, digging in extremely hard formation, has dug approximately 3,500,000 cu yd per yr. Another, with 18-cu-ft buckets digging to 124 ft below water level, has averaged over 4,000,000 cu yd annually for more than 3 years. A dredge of the same bucket capacity but digging only 80 ft below water level will dig under normal conditions from 15% to 20% more. Deeper digging presents operating conditions which reduce the yardage dredged. In the Hammonton area on the Yuba River in California, where the deep digging dredges are operating, approximately 900,000,000 cu yd have been dug. In the Folsom area on the American River in California, the total yardage dug is over 850,000,000 cu yd. The operating costs, including operating labor and supplies, power, repair labor and materials, and replacement parts usually have been 5¢ per cu yd or less for large placer dredges.

The two bucket-ladder dredges designed for use on the Panama Canal sea-level project incorporate a wealth of experience. All units have been designed for easy and quick replacement and for long service. Manufacturers of the various parts—buckets, gearing, electric and power equipment—were consulted. After a prolonged study, buckets of 54-cu-ft capacity were recommended. Buckets larger than this create many difficult problems in other machinery units making up the dredge, and this fact suggested limiting the designs to dredges having bucket capacities of 54 cu ft.

The dredge which digs 90 ft below water level will have units interchangeable with the deeper digging dredge, except for such units as the hull, superstructure, etc. The hull for the dredge digging 90 ft below water level will be approximately 245 ft long, 100 ft wide, and 15 ft deep; for the dredge digging 145 ft below water level, the hull will be approximately 371 ft long, 100 ft wide, and 13 ft deep. The installed horsepower will be 4,900 and 5,294, respectively. For the dredge digging 90 ft, the power plant will consist of three 1,600-hp diesel engines, each direct connected to a 1,250-kva, 2,300-volt generator. Four of the same units will constitute the power plant of the deeper digging dredge.

The digging ladder for the dredge digging 90 ft will be 163.5 ft between tumbler centers. For the deeper digging dredge the ladder will be 239 ft long. The buckets will have a 6-ft pitch and a height of 86.5 in. At the base the buckets will be 60.5 in. wide; at the lips, 74 in. There will be 62 and 88 buckets, respectively, on these dredges—the maximum bucket speed being 15.5 buckets per min.

The disposal system will provide for dumping the material from the buckets into the hopper, diverting them either to port or starboard, into barges. The weight of the dredge which digs 90 ft is estimated at approximately 5,900 tons, and that of the deeper digging dredge, approximately 7,200 tons. These weights do not include live loads.

In determining the estimated yardages and costs, dredges operated in deep and hard digging were considered. Although the maximum bucket speed will be 15.5 buckets per min, a speed of 12 buckets per min was used in the calculations. It is estimated that the dredge digging 90 ft will dig approximately 5,400,000 cu yd per yr and the deeper digging dredge, approximately 4,327,000 cu yd per yr.

Included in the calculations of costs were operating labor, operating supplies, power, repair labor, repair materials, and replacement parts. The salaries used were those in force in the Canal Zone under the schedule of July, 1946, including a sick leave of 20%. A work week of 6 days was considered, and a power cost of 2¢ per kw-hr. On this basis the estimated unit operating cost is slightly more than 20¢ per cu yd for the deep digging dredge, and slightly more than 15¢ per cu yd for the dredge to dig 90 ft.

In conclusion, it is to be noted that harbor dredges have been using buckets as large as 54-cu-ft capacity, and placer mining dredges have worked as deep as 130 ft below water level. Therefore, the dredges described and recommended for the Panama Canal combine the practices of both types of dredging and do not present any insurmountable difficulties.

Raphael G. Kazmann,[101] Assoc. M. ASCE.—An extremely valuable published addition to technical knowledge of some of the possibilities of relocating and redesigning the Panama Canal is presented in this Symposium. However, it is difficult, if not impossible, for an engineer not directly connected with the canal studies to criticize the application of the techniques described therein, simply because criticism implies differing experience on projects of the same order of magnitude in the same, or closely allied, fields. Such projects are few in number. Furthermore, engineers may criticize components of the findings in the Symposium (such as the applicability of soil mechanics, the navigational problems caused by tides, or the design of special dredges) without affecting the over-all correctness of the findings.

The converse is also true: Although the technical excellence of the methods is accepted, the very basis of the project may be so weak that technical excellence is a purely academic matter in determining its feasibility or desirability. The Symposium has clearly shown that engineering techniques now make possible a sea-level Panama Canal. The real question is, is one needed? The Symposium assumes that one is and hardly bothers with the point. It is possible, however, that the point is debatable and that someone who has not been as close to the details of the problem as the authors may be able to suggest a fresh viewpoint.

It is well to emphasize that the security of the canal is only one facet of the problem of national security. Others have already pointed out that in 1942, when the national security was actually in jeopardy, Panama Canal improvements could not complete with other projects for men and materials although more than $75,000,000 of a total of $277,000,000 had already been invested in the third locks "insurance" program. It should be remembered

[101] Hydrologic Engr., Ranney Method Water Supplies, Inc., Columbus, Ohio.

that certain Tennessee Valley Authority dams were still under construction and that all manner of large scale engineering construction projects, including a "fantastic" investment in the production of atomic energy, were being undertaken at the very moment when the canal improvements were being stopped because they were not vital to national security. If the canal were as necessary as the sea-level canal proponents claim, the third set of locks should and would have been completed during World War II. Surely the strategic situation in 1948 is far more favorable than it was in the dark days of 1942. Thus, a sea-level canal is probably needed less at present than a third set of locks was needed in 1942, and apparently at that time it was not needed at all.

It should be remembered that the United States has only so much man power and material at its disposal for construction projects. At present the labor and materials shortage is well known to every engineer. The expenditure of $2,500,000,000 (minimum estimate) over a 10-year period would be enough to accomplish the unified development of, for instance, the entire Missouri Valley Basin, thus increasing the physical resources of the country by 5,000,000 irrigated acres and by about 5,000,000,000 kw-hr per yr of firm power—in addition to preventing the damage due to droughts and floods. Moreover, about one third of the expenditure would be repaid within 50 years through the sale of power and water. The flood-control benefits and navigational improvements would account for the rest of the expenditures.

The question before the public in general, and the engineering profession in particular, when dealing with engineering proposals of this order of magnitude, is: As the United States has only so much man power and materials, is national survival made more secure by doing this or that, or the other, inasmuch as it is manifest that all cannot be done? A project of such magnitude cannot stand solely on its internal merit; it must stand on its utility as compared to other projects of the same magnitude and of equal internal merit, when viewed objectively in the national and international framework.

A sea-level Panama Canal seems to rate far down the list of desirable projects from the standpoint of national security. The unified development of the valley of the Tennessee River made possible the atomic energy program. The development of other basins will make possible a higher standard of living and may form the material basis for similarly great technical advances. The most potent construction equipment of the United States was designed in the course of river basin development, not only on the Tennessee Valley but also on the great flood-control and reclamation projects in the west. Similarly, the great advances in techniques of soil mechanics and hydraulic design resulted from river basin development. The further development of part of the internal transportation network—for example, the joining of the Ohio River and the Great Lakes-St. Lawrence River systems, and the improvement of the latter—could be fairly compared with the sea-level canal project. Which of these would to a greater degree improve the national ability to win a war?

It should be borne in mind at all times that military technology is based on the nonmilitary material and technological resources of a country. Would the nation be more likely to win a war with a developed Missouri Basin and

an improved lock canal as compared to an undeveloped Missouri Basin and a sea-level canal? Similarly, compare a fully harnessed Columbia River, a unified and operating St. Lawrence and the Ohio River waterway system, a completed Central Valley project, a complete new major road net, urban redevelopment, or completed atomic energy power plants with the utility of a sea-level canal. Which of these would be more important to national existence in the event of a conflict? Which would add more to the striking power of the United States? These are the alternatives that an engineering symposium may fairly be expected to mention, thus putting the proposed technical studies into a proper perspective rather than giving the impression that the issue had already been decided in favor of a sea-level canal and that the question is solely one of technique. Thus, the highest type of professional judgment could be brought to bear on proposed engineering projects, evaluating them as objectively as possible, both as to technical details and as to the possible alternatives of accomplishing the prime objectives—national strength and security. This procedure would assist the public and its duly elected representatives in arriving at a proper decision.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

MATRIX ANALYSIS OF PIN-CONNECTED STRUCTURES

Discussion

BY A. FLORIS

A. FLORIS.[7] ESQ.—In an abstract mathematical form the author develops an elegant method of analyzing stresses in framed structures. The concise presentation of the subject, coupled with the advanced technique used by the author, renders the understanding of the paper rather difficult. Nevertheless, the method is distinctly new and interesting, and as such should be appreciated.

The proposed theory follows paths opposite to those in common use. The true displacements of the bars are first determined, and the stresses in the bars are determined afterward. In the standard methods the opposite is true.

The problems with which the author illustrates his theory are solved much easier by the well-known methods. In Example 2, for instance, one equation is needed to determine the stress in the hyperstatic bar of the frame (Fig. 3) instead of the formidable matrix α (Eq. 17*c*). Using the standard methods, Example 1, is more tedious than Example 2, since it involves the solution of two simultaneous equations. If the author's method is applied, the solution of Example 2 is more complicated than the solution of Example 1. In hyperstatic structures of high degree of redundancy, the proposed method will show perhaps its superiority over the older methods. It is left to the discretion of the author to demonstrate this superiority in his closing discussion.

The elements of matrix for planar frames (Eq. 7) and those of matrix for space frames (Eq. 21) are symmetrically arranged about their main diagonal—thus showing a striking analogy to the elements of the determinants of the elastic equations obtained by other methods.

By removing the hyperstatic constraints or the bars of the structure and by applying the principle of virtual displacements or Castigliano's theorems, the the elastic equations are derived. The coefficients of the hyperstatic quantities are usually of the form—

NOTE.—This paper by Pei-ping Chen was published in December, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1948, by Rufus Oldenburger.

[7] Los Angeles, Calif.

$$\begin{vmatrix} \alpha^2 & \alpha\,\beta & \alpha\,\gamma & \cdots \\ \alpha\,\beta & \beta^2 & \beta\,\gamma & \cdots \\ \alpha\,\gamma & \beta\,\gamma & \gamma^2 & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{vmatrix}$$

—which is the common denominator of the hyperstatic quantities.

The elements in the foregoing determinant represent virtual displacements, whereas the elements in Eqs. 7 and 21 are pure coordinants.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

REVIEW OF SLOPE PROTECTION METHODS

REPORT OF THE SUBCOMMITTEE ON SLOPE PROTECTION OF THE COMMITTEE ON EARTH DAMS OF THE SOIL MECHANICS AND FOUNDATIONS DIVISION

Discussion

By Howard J. Hansen, William P. Creager, Henry H. Jewell, Joe W. Johnson, and Martin A. Mason

Howard J. Hansen,[13] Assoc. M. ASCE.—It is true that specific design data for the protection of slopes by riprap and other methods are meager and that there is a definite need for experimental research in this field. Certain formulas and procedures for computing wave heights, lengths, and velocities are presented in this excellent review of slope protection methods, on which the writer offers the following comments:

Eq. 6 is a formula used by engineers of the Tennessee Valley Authority to compute the equivalent current velocity, v, of a wave such as may be generated in a fresh water reservoir. This formula is similar in form to that derived by the late David A. Molitor, M. ASCE. The terms of his original formula—

$$v = 2.26\,c\,\sqrt{L} + 7.11\,\mu\,\frac{h}{\sqrt{L}} \qquad (9a)$$

—can be rearranged to give

$$v = \left[2.26\,c + 7.11\left(\frac{h}{L}\,\mu\right)\right]\sqrt{L} \qquad (9b)$$

Inasmuch as this formula, in general, gives somewhat higher values for the equivalent current velocity than Eq. 6, it is believed that the value of the paper could be enhanced by including an explanation of those factors which were added to Mr. Molitor's formula in determining Eq. 6.

The formula derived by Mr. Molitor, and which forms the basis for Eq. 6, is based on the trochoidal theory of waves first developed by Franz Gerstner.[14,15]

Note.—This report was published in June, 1948, *Proceedings*.

[13] Prof. of Mechanics and Acting Head of Industrial Eng. Dept., Univ. of Florida, Gainesville, Fla.

[14] "Theorie der Wellen—Abhandlungen der Koniglichen Bohmischen Gesellschaft der Wissenschaften," by Franz Gerstner, Prague, 1802.

[15] *Annalen der Physik*, Vol. XXXII, 1809, pp. 412–445.

This theory is based on rotational flow and does not consider the possibility of the wave movement being accompanied by a mass transport in the direction of wave travel.

The irrotational theory of G. G. Stokes,[16] T. Levi-Civita,[17] and D. J. Struik[18] is based on the fact that the waves are not sinusoidal in form and that there is an induced drift or mass transport in the direction of wave motion which modifies the wave velocity and orbital motion. This theory has been checked experimentally,[19] and it seems, therefore, that Eq. 6 should be based on the irrotational theory, although it is possible that the addition of the mass transport velocity to Eq. 6 would be sufficiently accurate. A summary of the equations based on the irratotional theory may be found in several publications.[20,21]

WILLIAM P. CREAGER,[22] M. ASCE.—The chairman and members of the Subcommittee on Slope Protection have made a very commendable study and report on the work assigned to them. The writer, in behalf of the Committee on Earth Dams, wishes to thank them for this excellent contribution.

Any report on a subject as controvertible as this one is will bring forth some differences of opinion. The fact that such differences will be included in some of the discussions, including that of the writer, does not detract at all from the great value of the report which has, for its primary purpose, the consolidation of ideas on the subject.

In the design of dumped riprap, one is interested in both the required depth of riprap and the required size of stone included therein. It will be noted that all the equations and diagrams in the report are for the purpose of deriving the size of stone—that is, the "equivalent sphereical diameter" (termed also the "theoretical stone size") necessary to resist movement by waves. It is not until tentative specifications for dumped riprap are given that the necessary depth of riprap is mentioned.

In these specifications the required depth of riprap is defined as the depth required to accommodate the theoretical size stone, with a tolerance in surface elevation designed to permit an occasional oversize stone. This would indicate that, with a certain unavoidable tolerance, it can be assumed that the specified depth of riprap should correspond to the calculated theoretical size of stone. This is also indicated in Table 1, in which the recommended depth of riprap is reasonably close to the theoretical size for the 155-lb stone.

It will be noted that the recommended depth of riprap (Table 1) is independent of the unit weight of the stone and, presumably, it agrees with the

[16] "On the Theory of Oscillatory Waves," by G. G. Stokes, *Transactions*, Cambridge Philosophical Soc., Vol. VIII, 1847, p. 441 and Supplement; *Scientific Papers*, Vol. 1, p. 314.

[17] "Détermination Rigoureuse des Ondes d'Ampleur Finie," by T. Levi-Civita, *Mathematische Annalen*, Vol. XCIII, 1925, pp. 264–314.

[18] "Détermination Rigoureuse des Ondes Irrotationelles Periodiques dans un Canal à Profondeur Finie," by D. J. Struik, *ibid*, Vol. XCV, 1926, pp. 595–634.

[19] "A Study of Progressive Oscillatory Waves in Water," *Technical Report No. 1*, Beach Erosion Board, Office of the Chf. of Engrs., Washington, D. C., 1941.

[20] "A Summary of the Theory of Oscillatory Waves," *Technical Report No. 2*, Beach Erosion Board, Office of the Chf. of Engrs., Washington, D. C., 1942.

[21] "Beach Erosion Studies in Florida," by Howard J. Hansen, *Bulletin No. 16*, Florida Eng. and Industrial Experiment Station, Gainesville, Fla., 1947.

[22] Cons. Engr., Buffalo, N. Y.

theoretical size of the lightest (155-lb) stone purely for conservatism. To bring out this feature, the writer has plotted values from Table 1 in Fig. 7, which shows, by the full lines, the theoretical size stone for the three unit weight stones and, by the dotted line, the recommended depth of riprap. It will be noted, again, that the recommended depth of riprap corresponds to the theoretical size for a 155-lb stone.

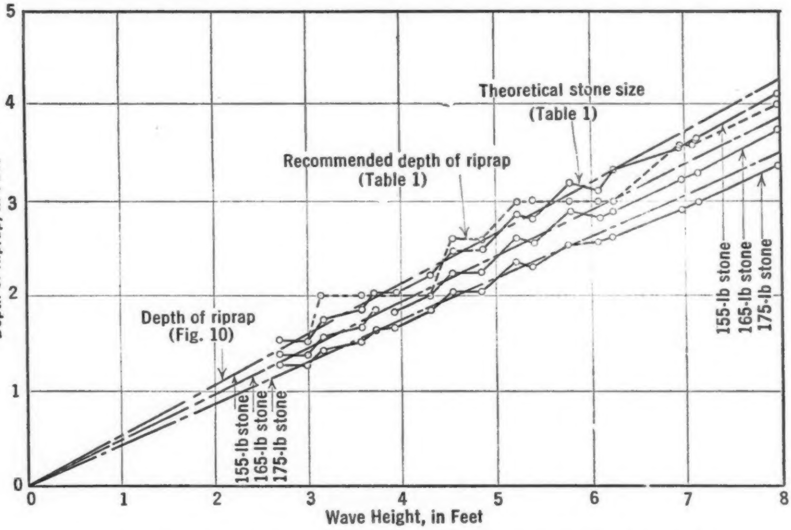


FIG. 7.—PLOTTING OF VALUES FROM TABLE 1 FOR DAM SLOPE OF 1 ON 3

Based on the foregoing assumption, the writer has prepared Fig. 8, from which the required depth of riprap can be determined more readily than by the use of the several illustrations in the report. Fig. 8 has been derived as follows: It was found that the elimination of the condition imposed by Fig. 3, between values of wind velocity of 50 miles per hr and 100 miles per hr, made very little difference in the required depth of riprap. Omitting this condition, the required depth of riprap, for a given dam slope and unit weight of stone, is dependent only on wave height. Fig. 8 for 165-lb stone has been calculated from the equations and curves in the report, with that condition omitted. Surprisingly, the resulting diagram consists of straight lines.

Fig. 8 is for a unit weight of stone of 165 lb per cu ft. The required thickness for other unit weights of stone may be obtained from

$$d_s = \frac{102.5\, d_{165}}{s - 62.5} \qquad (10)$$

which was readily derived from Eq. 5 and its equation for the value of Z. It will be noted that the required thickness varies with the submerged unit weight of stone. In Eq. 10, d_s is the required thickness for stone having a unit weight of s pounds per cubic foot, and d_{165} is the thickness required for $s = 165$, obtained from Fig. 8.

Fig. 8 gives depths of riprap that agree with the report methods within 4% of the riprap depth, which is well within the accuracy of the basic data. In order to indicate the accuracy of Fig. 8, the required depth of riprap, for a dam slope of 1 on 3, obtained from Fig. 8 and Eq. 17, has been indicated in Fig. 7 by dot-and-dash lines for three values of unit weight of stone. It will be noted that they agree with the report values within the accuracy stated for the same unit weight of stone—the only difference being that due to the omission of the condition imposed by the use of Fig. 3, as previously mentioned.

In Fig. 8, the slopes of 1 on 1 and 1 on $1\frac{1}{2}$ have been omitted because such slopes are too steep where considerable wave action is possible. A slope of 1 on 12 is seldom used, and the depths recommended in the report appear relatively excessive for such a flat slope. However, it has been included in Fig. 8 solely for further discussion.

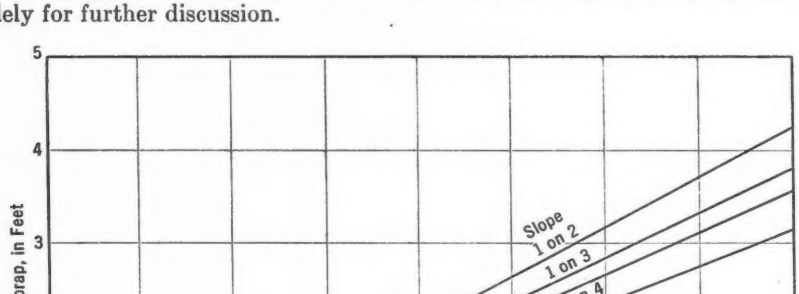


FIG. 8.—RELATION BETWEEN WAVE HEIGHT AND DEPTH OF RIPRAP FOR STONE WEIGHING 165 LB PER CU FT

Fig. 8 is based on the writer's interpretation of the recommended depth of riprap for various wave heights as given in the report. Because of a lack of experiments and observational data it is not possible to criticize its recommendation, since successful riprap of about the same thickness has been used. However, the difference between the riprap thickness for the 1 on 2 and the 1 on 4 slope, and particularly between the 1 on 2 and the 1 on 12 slope, appears very small indeed. Experimental data are badly needed.

With reference to the size of stone composing the riprap, the tentative specifications in the report require that 50% by weight should consist of stone equal to or larger than the theoretical size, with a few larger stones, up to about twice the weight of the theoretical size, tolerated for reasons of economy in the utilization of the quarried rock.

This specification must contemplate processed stone, because, for the usual run of well-graded quarried rock, the maximum size is about twice as large (equivalent spherical diameter) as the 50% size with about eight times its weight. Therefore, if, as is customary, quarry run rock is used and 50% by weight of the stones are equal to or larger than the theoretical size, then instead of there being a "few" larger stones, there would be 50% by weight larger stones which, instead of being limited to "twice," would range up to eight times the weight of the theoretical size. The average depth of the riprap would be much greater than that shown in Col. 15, Table 1.

There are large elements of conservatism in the experiments of S. Ishbash upon which the equivalent spherical diameter of stone given in the report is based. These elements are:

1. The stones in the Ishbash experiments were dropped in flowing water.
2. They were not at rest and were not well nested.
3. The velocity of the water was directed down the slope and hence the movement of the stones was assisted by a component of gravity.
4. The velocities used in the report are directed up the slope and a component of gravity assists stability.
5. The actual velocities down the slope of dams, as the wave recedes, are much less than they are up the slope.

For this reason the writer feels that 50% of the stone by weight need not be equal to the calculated equivalent spherical diameter. It is his opinion that the specifications should call for stones which are reasonably well graded, and with the maximum size having a weight equal to the weight of the theoretical size stone—that is, having an equivalent spherical diameter equal to the required depth of riprap. If this is adhered to, then 50% of the stones by weight would have an equivalent spherical diameter of about one half the maximum, a weight of one eight of the maximum weight, an average least dimension of about one third of the maximum equivalent spherical diameter, and a longest dimension of about one half of the maximum equivalent spherical diameter.

When the fetch is not normal to the center line of the dam, the waves will strike the riprap at an angle. In this case the resistance of the stones to movement will be less than that when the waves are normal and the thickness should be increased somewhat. The empirical equation—

$$d' = d\left(1 + \frac{\sin 2\,\alpha}{4}\right) \qquad (11)$$

—is offered, in which d' is the thickness required for the angle; d is the thickness required for $\alpha = 90°$; and α is the angle between the direction of the wind and the center line of the dam.

Eq. 11 gives a depth of 1.0 d for $\alpha = 90°$ and $\alpha = 0°$, and a maximum of 1.25 d when $\alpha = 45°$. Surveys of the efficacy of existing riprap protection and experiments with wave tanks, both of which are under way, will throw additional light on the problem, and the results will undoubtedly be made available in subsequent reports of the subcommittee.

The report states that, on several occasions, wave heights have been reported which greatly exceeded the Stevenson-Molitor formula (Eqs. 2) and the Creager formula (Eq. 5) with $C = 3.41$. These examples are not mentioned in the report. A much quoted example is that given for the Ashokan Reservoir of the New York (N. Y.) water supply. The writer has investigated this and other examples of excessive wave heights and has reached the conclusion that the damage attributed to such waves was mostly due to wind blown spray. This spray shoots up the face of the dam to considerable heights and it is usually more economical to protect the top of the dam from its erosive action than to raise the dam. The writer has also investigated the results of the research of the Scripps Institution of Oceanography at La Jolla, Calif., on wave heights and published assembled data from that research, and believes that it is not applicable to inland waters.

The report states that the condition of some old installations of hand-placed riprap was such as to justify the conclusion that hand-placed material is no more effective than an equal depth of dumped riprap. The writer cannot subscribe to this conclusion and wonders if the damage to the old hand-placed riprap was not caused by an inadequate filter blanket. Filter blankets placed in one layer are hazardous unless the material is especially well graded and uniform in the borrow pit and unless the inspection is exceptional. The use of even a very small area protected with an inadequate filter obtained from a defective place in the borrow pit will provide the entering wedge of wave action, which will rapidly extend by undermining to other well protected areas. The requirement of layers screened to specifications will insure the proper gradation.

The writer has had sad experience in the use of pipe drains for berms of dams because they become plugged when gullying occurs during the first stages of protective plant growth. Open paved gutters down the slope are preferred.

Little has been stated in the report relative to the use of sand or gravel for protecting the downstream slope. The downstream surface of the Soft Maple Dam in northern New York State is composed of sand with an effective size of about 0.17 mm. After 20 years of service the heaviest rains have been absorbed and no gullies have appeared.

Where a gravel blanket is placed on an impervious fill, the rainfall will not be absorbed by the dam and must run down the slope through the interstices of the gravel. If the gravel layer is too thin, it will become saturated and tend to slough. The thickness should be based on the rate at which the rainfall can be conveyed to the bottom of the slope, depending upon the porosity of the layer, the slope, and the height of the dam.

HENRY H. JEWELL,[23] M. ASCE.—A very interesting summary of valuable information relative to slope protection of earth dams has been presented in this report.

Some advantage may result from grouting rock riprap carefully placed by hand on an adequate filter bed subjected to moderate wave action. Grouting dumped riprap or hand-placed riprap on earth dams subjected to heavy wave

[23] Asst. Chf. Engr., Federal Works Agency, Washington, D. C.

action as a rule is effective only for a relatively short period and is a wasteful expedient. It adds little to the durability of the protection. Cracks in the grout are inevitable and the resulting damage becomes progressive. The extensive cracking which generally follows of course makes the grout ineffective. If the underlying filter bed is inadequate, the fine materials will pass out through the cracks under wave action.

Grouting dumped riprap was tried in the early riprap repairs to the concrete block protection on the Kingsley Dam in Nebraska, under the theory of the proponents that it would "increase the structural strength" of the riprap blanket. Like the grouted concrete blocks, the grouted riprap failed to withstand heavy wave action. "Structural strength," similar to the strength characteristics of a structural slab, is unattainable by grouting dumped rock riprap on earth dams. Any such attempt tends to oppose the prime functions of rock riprap which are not only to serve as a protective cover but also to break up wave action and, with the underlying bed, to act as a filter.

Porous concrete is unsatisfactory as protection against wave action on earth dams not only because of deterioration caused by chemical and erosive forces but also because of its porous nature which makes it impracticable to reinforce the concrete to resist temperature stresses and to prevent cracking. Such use therefore violates an elementary fundamental principle of concrete slab paving. No experienced engineer would consider using a monolithic concrete slab without reinforcement for earth dam slope protection. Likewise, there is no sound basis to support the theory that an unreinforced porous concrete slab would successfully resist cracking. The experience at the Santee-Cooper project in South Carolina, where very serious and extensive cracking occurred and where 480,000 sq yd of porous concrete paving 10 in. thick (erroneously stated as 8 in. in the report) on 12 miles of dams are being replaced by dumped rock riprap, provides ample proof of the unsuitability of porous concrete for slope protection of earth dams.[24]

It is not always practicable or economical to adopt all the refinements indicated in Fig. 6 and in the section on "Filter Blankets: Design of Filter Blankets." It is impossible to place a filter bed of several relatively thin layers of graded materials on the slope of an earth dam in such a manner as to secure the full effect of the design. Some disturbance, impactment, and inaccuracies of grading and thickness in the process of placing are unavoidable. Under most conditions, ample thickness, of one layer of a material graded within reasonable limits of the gradation curve, will give satisfactory results. Great exactness in gradation and thickness and number of layers, if possible, can generally be secured only at prohibitive cost.

Joe W. Johnson,[25] M. ASCE.—The various problems involved in the protection of the slopes of earth dams are reviewed in this report, and suggested procedures for the designer to follow are presented. The section of the report which proved of interest to the writer is that concerning the theory of wave action and the application of this theory to slope protection design.

[24] "Rock Riprap Replaces Porous Concrete Slope Protection at Santee-Cooper Project," by Henry H. Jewell, *Civil Engineering*, January, 1948, p. 14.

[25] Associate Prof., Mech. Eng., Univ. of California, Berkeley, Calif.

No mention is made in the report of the extensive theoretical and experimental work which has been published during the decade from 1938 to 1948 on oscillatory waves in general, and on the characteristics of wind generated waves in particular. Instead, a few empirical relationships of doubtful value constitute the basic design equations. The most complete up-to-date summary of the theory of oscillatory waves that is available to the engineer is that compiled in 1942 by the Beach Erosion Board.[20] Thus, for waves of small amplitude the wave length and the wave velocity are related to depth by

$$v = \sqrt{\frac{g\,L}{2\,\pi}\tanh\frac{2\,\pi\,d}{L}} \qquad (12)$$

Also, the relationship between period, length, and velocity characteristics for all wave phenomena is

$$L = v\,T \qquad (13)$$

in which, conforming to the notation in the report, v is the velocity of the wave form, in feet per second; L is the wave length, in feet; d is the water depth, in feet; and T is the wave period, in seconds. Eq. 12 applies to waves in both deep water and shallow water. The validity of this equation has been verified experimentally in both salt and fresh water, and in deep and shallow water.[19,26] Agreement between theory and experiment in most instances is within 2%. The distinction between deep-water waves and shallow-water waves as used herein is that deep-water waves exist when the water depth is greater than half the wave length—that is, when d becomes greater than $L/2$, the term, $\tanh 2\,\pi\,d/L$, in Eq. 12 approaches unity and the velocity expression becomes

$$v_0 = \sqrt{\frac{g\,L_0}{2\,\pi}} \qquad (14)$$

However, as

$$L_0 = v_0\,T \qquad (15)$$

$$v_0 = \frac{g\,T}{2\,\pi} = 5.12\,T \qquad (16)$$

and

$$L_0 = 5.12\,T^2 \qquad (17)$$

The subscript zero in Eqs. 14, 15, 16, and 17 refers to deep-water conditions. It is noted that the wave velocity and wave length in deep water are simple functions of wave period. In practice it is usually more convenient to use the wave period rather than the wave length, especially where shallow-water waves are considered, because the period is a constant and is relatively easy to observe. For the majority of problems on wave action in reservoirs only deep-water conditions are important.

It should be noted that Eq. 14 gives only the velocity of the wave form. The water particles follow approximately a circular path in deep water during

[26] "Breakers and Surf: Principles in Forecasting," *Publication No. 234*, Hydrographic Office, U. S. Navy, Washington D. C., 1944, p. 6.

the passage of a wave. The radius of the orbits varies from a maximum at the surface to practically zero at a depth equal to half the wave length. At the water surface the diameter of the orbital path is a maximum and is equal to the wave height. Consequently, the maximum orbital velocity in a vertical element is at the water surface in deep water and is given by

$$u_{max} = \frac{\pi h}{T} \qquad (18)$$

in which h is the wave height, in feet.

It is not clear whether Eq. 6 as recommended by the subcommittee for computing the wave velocity is comparable to Eq. 14 for the velocity of the wave form, or to Eq. 18 for the velocity of the water particles. Although the subcommittee refers to the velocity computed from Eq. 6 as a "current velocity" and applies the term to the transportation of sediment where water velocity must be considered, it appears from the report (last paragraph under the heading, "Destructive Forces That Act on Embankment Slopes") that Eq. 6 is intended to give the velocity of the wave form. The departure of the value of velocity computed by Eq. 6 from that value computed by Eqs. 14 and 18 is discussed hereinafter.

The main source of information on the relationships between wave characteristics and a generating wind, as adopted by the subcommittee, is the observational data of Thomas Stevenson[1] in 1874 and of D. D. Gaillard[2] in 1904. Engineers engaged in studies on wave action should become familiar with the theoretical relationships developed during World War II by H. U. Sverdrup and W. H. Munk.[27] Using these theoretical relationships and all the reliable observational data that were available Messrs. Sverdrup and Munk developed a set of curves yielding the relationships between the height and period of waves generated by a wind of given speed and duration blowing over a given fetch.[28] Curves are also given for the minimum duration time required to raise the highest wave for a particular fetch. These curves were used extensively during World War II to forecast sea, swell, and surf conditions for amphibious operations.[29] Because of considerable observational data which have become available by the installation of wave recorders during and after World War II, some recent revisions to the wave forecasting graphs have been made.[30] The wind data used in determining wave characteristics from the forecasting graphs can be obtained either from actual wind observations or from weather charts. Mention is made by the subcommittee that the Sverdrup-Munk curves have been applied to the estimation of wind waves on San Francisco Bay (Cali-

[1] "The Design and Construction of Harbours," by Thomas Stevenson, A. C. Black, Edinburgh, 1874.

[2] "Wave Action in Relation to Engineering Structures," by D. D. Gaillard, *Professional Paper No. 31*, Corps of Engrs., U. S. Army, U. S. Govt. Printing Office, Washington, D. C., 1904.

[27] "Wind, Sea, and Swell: Theory of Relations for Forecasting," by H. U. Sverdrup and W. H. Munk, *Publication No. 601*, Hydrographic Office, U. S. Navy, Washington, D. C., March, 1947.

[28] "Wind Waves and Swell: Principles in Forecasting," *Miscellaneous Publication No. 11,275*, Hydrographic Office, U. S. Navy, Washington, D. C., 1944.

[29] "Breakers and Surf: Principles in Forecasting," *Publication No. 234*, Hydrographic Office, U. S. Navy, Washington, D. C., 1944.

[30] "Revised Wave Forecasting Graphs and Procedure," *Wave Report No. 73*, Scripps Institution of Oceanography, Univ. of California, La Jolla, Calif., March, 1948 (mimeographed).

fornia),[4] but the application of the forecasting curves to inland waterways is questioned by it.

The reliability of the wave forecasting curves and techniques is verified in papers by R. Stump[31] and J. D. Isaacs and Thorndike Saville, Jr.,[32] Jun. ASCE, in which observed wave heights and periods are compared with those determined by the forecasting methods. The paper by Messrs. Isaacs and Saville, in particular, gives convincing information on the reliability of the Sverdrup-Munk curves, in addition to a comparison between forecast wave characteristics and those wave conditions obtained from wave recorders operated by the Department of Engineering at the University of California (Berkeley), in cooperation with the United States Navy Bureau of Ships. These recorders have been in operation since 1947 at Point Sur in California, and Heceta Head in Oregon. A statistical analysis of forecast and observed data showed that 97% of the recorded significant increases in wave height were forecast. Of the arrival times, 69% were predicted within 6 hours and usually were predicted earlier than those actually occurring. Of the forecast wave heights, 53% were within 1 ft of the recorded heights; and 80%, within 2 ft. The forecast periods were usually lower than the recorded periods, 63% being within 2 sec of the recorded periods. In general, it can be stated that the forecasting technique results in a high degree of reliability for forecasting the arrival of significant increases in wave height, and prognosticating the wave heights. These two factors are important in controlling marine and shore activities.

There appears to be little question as to the accuracy of the Sverdrup-Munk curves in calculating wave heights and periods (or wave lengths) for relatively large generating areas such as those found in oceans. To provide data on the applicability of the Sverdrup-Munk curves to lakes and protected bays where fetches are relatively small, observations were made by the University of California at Clear Lake, Calif., in 1944 and again in 1946.[33] These investigations were preliminary in character and were intended to serve as a guide to a more complete investigation. Data on the variability of wave length and crest length as a function of fetch in a generating area were obtained in these tests, however, and observed wave heights and periods were of the same order of magnitude as those given by the Sverdrup-Munk curves. Following the Clear Lake tests, observations on wind and wave characteristics were made in the fall of 1947 at Abbotts Lagoon on Point Reyes in California. This lagoon is a body of fresh water approximately 3,000 ft long by 1,000 ft wide and so oriented that the prevailing winds in that locality blow along the longer dimension of the lagoon. Relatively low sand dunes surround the lagoon so that effects of topography on wind distribution probably are negligible. A wave recorder was located at the down-wind end of the lagoon. Wind speed and direction recorders were located at both ends of the lagoon, wind speed near

[4] "Estimating Storm Conditions in San Francisco Bay," by J. A. Putnam, *Transactions*, Am. Geophysical Union, April, 1947.

[31] "Forecasting Waves and Surf," by R. Stump, *Shore and Beach*, Vol. XVI, No. 1, April, 1948, pp. 3-7.

[32] "A Comparison Between Recorded and Forecast Waves on the Pacific Coast," by J. D. Isaacs and Thorndike Saville, Jr., New York Academy of Science, March, 1948 (unpublished).

[33] "The Characteristics of Wind Waves on Lakes and Protected Bays," by J. W. Johnson, presented before the Annual Meeting of the Am. Geophysical Union, Washington, D. C., 1948.

the wave recorder being measured by a bank of cup anemometers located at elevations of $4\frac{1}{2}$ ft, 9 ft, 13 ft, and 17 ft above the water surface.

The Abbotts Lagoon data for those conditions, where the duration time of the wind exceeded the minimum time to raise the highest wave, were plotted

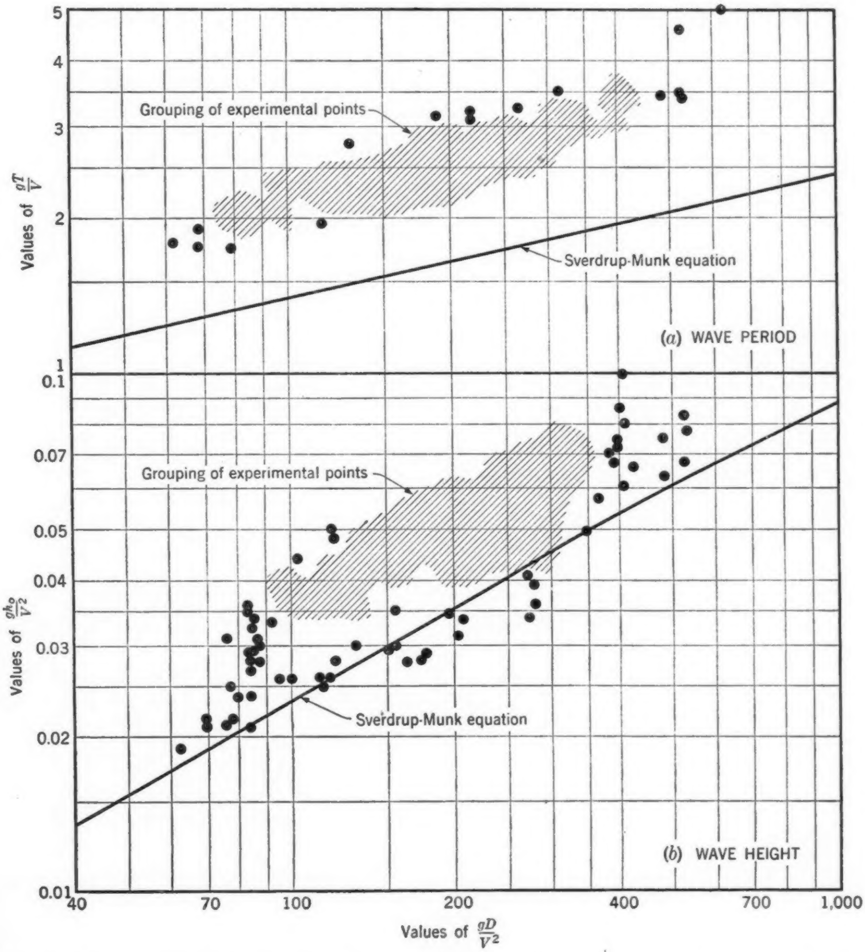


FIG. 9.—RELATIONSHIP BETWEEN WAVE PERIOD, WAVE HEIGHT, FETCH, AND WIND SPEED FROM OBSERVATIONS AT ABBOTTS LAGOON, CALIFORNIA, COMPARED WITH SVERDRUP-MUNK CURVES (WIND SPEED MEASURED $4\frac{1}{2}$ FT ABOVE THE WATER SURFACE)

using nondimensional parameters (Fig. 9). Fig. 9(*a*) shows a parameter containing wave period as a function of wind speed and fetch, whereas Fig. 9(*b*) shows a function of wave height as a function of the fetch-wind speed parameter. For comparative purposes the Sverdrup-Munk curves also are included in Fig. 9. To avoid confusion only data from a few days of observation at Abbotts Lagoon are shown. The various points, both for wave height and wave

period, lie above the Sverdrup-Munk curves. Some of this difference can be explained as a difference in the basic data used in the two analyses. Of perhaps greater importance is the fact that the wind speed used in the Abbotts Lagoon analysis was that measured by the anemometer located nearest the water surface (namely, at an elevation of $4\frac{1}{2}$ ft). As an appreciable wind gradient was observed to exist from measurements with the bank of anemometers, the use of the wind speed from the anemometer located $17\frac{1}{2}$ ft above the water would have made the Abbotts Lagoon data fall closer to the Sverdrup-Munk curves; for example, when a wind speed of 15.5 ft per sec was observed at an elevation of $4\frac{1}{2}$ ft, the wind speed at an elevation of 17 ft was 22 ft per sec. Although Messrs. Sverdrup and Munk are not too clear on where the wind speed should be measured, it usually is assumed to be measured at an elevation of approximately 30 ft. An important problem for further research is that of wind gradients over relatively rough surfaces—such as over lakes when waves exist. Such information is needed to correlate better the various data on wind generated waves. It should be noted that the wave height used in Fig. 9 refers to the average of the highest third of the waves present. This definition of wave height has been used because these higher waves are generally considered to be the most important for design purposes in engineering work.

Although not shown in Fig. 9, the observational data from Abbotts Lagoon cover the range of values of the parameter $g\,D/V^2$ from approximately 100 to 3,000. This lower limit would apply in the case where a wind speed of 63 miles per hr is blowing over a 5-mile fetch. A wind speed of only 30 miles per hr over a 5-mile fetch would give a value of $g\,D/V^2$ equal to 440. An upper limit of 3,000 for the value of $g\,D/V^2$ applies to a wind of 11.5 miles per hr over a 5-mile fetch, or a wind of 28 miles per hr over a 30-mile fetch. This range in values of the parameter $g\,D/V^2$ covers most of the conditions that normally would be encountered in reservoir problems.

Wave estimates made by the Sverdrup-Munk curves apply primarily to the two-dimensional problem. For a given fetch and a design wind whose duration time exceeds the minimum time to yield the maximum wave height, it is believed that estimates of wave characteristics by this relationship will be on the conservative side; that is, the effect of topography probably would so alter the wind pattern over a lake that lower heights would be produced than if a two-dimensional condition were assumed to exist. In no instance should a design wind speed that exceeds 60 knots be used with the Sverdrup-Munk relationships.

In view of the quantity and quality of the basic data used in the development of the Sverdrup-Munk relationships and with their verification in both the ocean[31,32] and at Abbotts Lagoon, it appears that the Sverdrup-Munk curves are superior to any of the formulas recommended by the authors for estimating wave heights. A comparison between these latter formulas and the Sverdrup-Munk curves is shown in Fig. 10, in which wave heights are plotted as a function of fetch for a wind speed of 50 miles per hr. This wind speed is used instead of the speed of 100 miles per hr used by the subcommittee, because it appears to be more in the range of conditions under which actual observations have been made. It is significant that the Sverdrup-Munk fore-

casting curves give no data for wind speeds greater than 60 knots. Observations on waves generated by winds at higher velocities than 60 knots are poor or nonexistent. Whether the mechanics of wave generaton at such high velocities is the same for lower velocities is still to be determined.

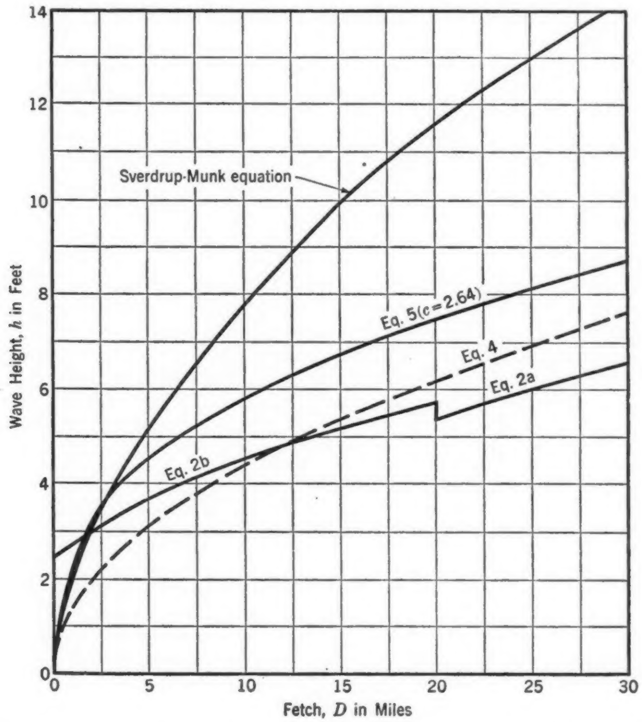


FIG. 10.—COMPARISON OF WAVE HEIGHT FORMULAS FOR v = 50 MILES PER HR

In presenting the comparison in Fig. 1 the subcommittee appears to have extrapolated formulas of doubtful value into a range of doubtful application. Fig. 10 shows that the Sverdrup-Munk curve generally yields relatively large heights compared to those computed from the other formulas. This may explain the statement in the report (under the heading, "Destructive Forces That Act on Embankment Slopes") that:

> "* * * experience has proved that its [the Stevenson-Molitor formula] results do not assure reliable indications of critical wave heights, several occasions having been reported in which wave heights have greatly exceeded those given by the formula."

One factor that may account for the relatively low wave heights observed in the past and used in the development of the earlier wave formulas is that the duration time of the wind may have been insufficient to raise waves to the maximum possible height.

The relationship for wave steepness (Eq. 3), as proposed by the late D. A. Molitor, M. ASCE, and used in the design procedure illustrated in Table 1, is of questionable value. Reference to Mr. Molitor's original statement shows that the ratio L/h varied between the limits 9.1 and 15. Furthermore, these values pertain to shallow-water conditions, and in most reservoirs deep-water conditions prevail practically up to the point where the waves break on the dam. A more accurate expression for wave steepness is that presented by Messrs. Sverdrup and Munk[34] wherein the height-length ratio (h/L) is plotted against wave age, which is defined as the ratio of wave velocity to wind speed (v/V).

The application of Eq. 3 as recommended in the report may give results of questionable value as illustrated by the computations presented in Table 2.

TABLE 2.—WAVE CHARACTERISTICS AS A FUNCTION OF WIND SPEED FOR A FETCH OF 25 MILES

V, in miles per hr	h, in ft (Eq. 2a)	h_0, in ft (Fig. 9(b))	$\frac{L}{h}$ (Eq. 3)	L, in ft (Eq. 3)	T, in sec (Fig. 9(a))	L_0, in ft (Eq. 17)	$\frac{L_0}{h_0}$ (Col. 7 ÷ Col. 3)	v_0, in ft per sec (Eq. 14)	v, in ft per sec (Eq. 6)	u_{max}, in ft per sec (Eq. 18)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
10	2.7	1.5	84	226	2.4	30	19.6	12.3	16.9	2
20	3.8	4.5	42	160	3.3	56	12.5	16.9	18.2	4.3
30	4.7	7.5	28	130	4	82	11	20.5	18.7	5.9
40	5.4	10.4	21	113	4.6	108	10.4	23.6	19.4	7.1
50	6	12.8	16.8	101	5.2	138	10.8	26.6	19.8	7.7

Thus, for a fetch of 25 miles, values of various wave characteristics are shown as a function of wind velocity as computed by the equations discussed herein and those recommended by the subcommittee. The formulas by which the various terms are computed are indicated in the column headings. Cols. 2 and 3 compare wave heights as computed by the Stevenson and by the Sverdrup-Munk formulas. A comparison of wave heights by all the various formulas has been discussed previously in connection with Fig. 10 and need not be repeated at this point. Cols. 4 and 5 show the wave steepness and wave length, respectively, as computed from a combination of Eqs. 2a and 3. It is noted from an inspection of Col. 5 that the wave length decreases with an increase in wind speed. This condition is in distinct disagreement with the established fact that wave length (or period), as well as wave height, increases with increasing wind speed for a given fetch. In Col. 6 values of wave period as calculated by the Sverdrup-Munk curve (Fig. 9(a)) are listed. Wave lengths computed by Eq. 17 using the wave period in Col. 6 are given in Col. 7. Compare these wave lengths with those shown in Col. 5. Also compare the values of wave steepness in Col. 4, as computed by Eq. 3, with those values in Col. 8. Other comparisons of interest are the wave velocities shown in Col. 9, as computed by Eq. 14 using the wave period in Col. 6, and the "equivalent current" velocity as calculated by Eq. 6. Shown in Col. 11 is the orbital velocity of the water

[34] "Wind, Sea, and Swell: Theory of Relations for Forecasting," by H. U. Sverdrup and W. H. Munk. *Publication No. 601*, Hydrographic Office, U. S. Navy, Washington, D. C., March, 1947, p. 15, Fig. 5.

particles at the water surface. Particles below the surface have orbital velocities which are less than the surface values.

As previously mentioned, the exact meaning of the value of the velocity given by Eq. 6 is not clear; however, it appears to be of the order of magnitude of the velocity of the wave form as given by Eq. 14. Although the water particles have a slightly greater velocity at the point of breaking than has the orbital velocity (Col. 11), it is doubtful if the velocity of water particles in a breaker will attain the values listed in Col. 10; but, as is stated in the report, it is the velocity of the water particles that must be considered in designing riprap. More details on the development and meaning of Eq. 6 would be of interest.

In most instances as waves move over the surface of a reservoir and approach a dam they are of the deep-water type. Because the upstream faces of most dams are relatively steep, the waves break as a plunging breaker with little time to undergo the transformations that usually occur when waves move shoreward over natural beaches. Most of the wave energy is dissipated in breaking; however, on steep slopes some energy of the waves may be reflected.

The forces acting on the face of the dam when waves break are more in the nature of impact forces,[35] rather than of a drag exerted by the tractive force of water flowing over a solid surface. Investigations in the wave tank at the University of California on the forces exerted on structures by wave action show that relatively high pressures exist for a relatively short time when a wave breaks (or even reflects) on a structure, and it is these forces occurring during each wave period which are probably responsible for the destruction of a riprap facing.

The tests of S. Ishbash[5] were concerned with the tractive force or the stream traction exerted in a steady state of flow on sediment resting on a stream bed. It is inconceivable to the writer that the forces exerted by stream traction and those exerted by the impact action of waves could be of the same order of magnitude. Mr. Ishbash's data hardly seem applicable to this problem. The conclusion of Mr. Gaillard as accepted by Mr. Molitor and as quoted in the report (under the heading, "Destructive Forces That Act on Embarkment Slopes") should be critically examined in the light of present-day developments. The apparent neglect of impact forces and the use of a formula which gives relatively low wave heights, as recommended by the subcommittee, both would contribute to the underdesign of the facing of dams.

In connection with the problem of wind set up, reference also should be made to the studies by B. Hellström in Sweden.[36] The wind effect on lakes and the shape of water surface are discussed in this publication and a theory is compared with actual observations on several European lakes and on Lake Erie and Lake Okeechobee in the United States.

---

[35] "Action and Effect of Waves," by J. W. Johnson and J. D. Isaacs, *Western Construction News*, April, 1948, pp. 97–102.

[5] "Construction of Dams by Dumping Stones Into Flowing Water," by S. Ishbash, Leningrad, 1932.

[36] "Wind Effect on Lakes and Rivers," by B. Hellström, *Proceedings*, The Royal Swedish Inst. for Eng Research, No. 158, Stockholm, 1941.

MARTIN A. MASON,[37] Assoc. M. ASCE.—The problem of downstream slope protection on earth dams can be said to be adequately understood and, at least to a reasonable degree, satisfactorily solved. Thanks are due to the soil conservationists and highway engineers for this circumstance. Methods of protection of earth slopes developed by these workers can be applied with little or no modification to the earth dam downstream slope problem.

In contrast, the basic elements of upstream slope protection on earth dams are imperfectly understood. They have been investigated or studied to an extent utterly incommensurate with the monetary and physical importance of the problem. Most upstream dam slope protection completed in the twentieth century must be considered as full-size experimentation; the report of the subcommittee confirms this belief.

It is certainly true, as the subcommittee report states, that the principal destructive forces acting on a dam face arise from wave action or the effects of wave action. However, the writer finds himself in disagreement with the subcommittee's analysis of the details of wave action and its effects on a slope. The following discussion will illustrate the points of disagreement.

For practical purposes, wave action in reservoirs can be considered as resulting wholly from wind action on the water surface. The important parameters with respect to the wind are the velocity, direction, duration, and variability of the wind structure. Duration implies consideration of the time during which given wind velocity and direction exist, as well as of the fetch, or distance in the direction of wind velocity over which the velocity and direction are essentially constant. The important parameter with respect to the water is the water depth. Configuration of the water surface of the reservoir probably is an important parameter for reservoirs having a high slenderness ratio, or a ratio of length of water surface along the axis to width. In these reservoirs the effects of diffraction and refraction of the waves may result in waves existing outside the area in which they are formed.

It is apparent, if the foregoing be true, that empirical formulas of the Stevenson-Molitor type do not represent true conditions and can be considered adequate for rough approximations only.

During the time of formation of waves in the generating area energy is transferred from the wind to the waves, the system of waves generated thus can be considered to carry a "package of energy" from the generating area to an area where the energy may be dissipated—namely, the upstream slope of the dam, or the banks of the reservoir. The major part of the energy is dissipated in the breaking of the waves on the slope, thus giving rise to destructive forces. These forces, which act individually and collectively on the obstacle causing the waves to break, are:

(*a*) A compressive force due to impact;
(*b*) A shearing force, directed upward, due to uprush of the wave on the slope; and
(*c*) A shearing force, directed downward, due to backwash of the wave down the slope.

[37] Chf., Eng. and Research Branch, Beach Erosion Board, Office of the Chf. of Engrs., Dept. of the Army, Washington, D. C.

The foregoing forces in turn result in the following phenomena:

(1) An impact over the area of wave break;
(2) Intermittent excess hydrostatic pressure of duration approximating half the wave period;
(3) A series of impulses, transmitted by the water in the interstices of the slope protection, producing internal pressures of short duration within the slope protection; and
(4) Alternate compression and expansion of air trapped in cavities or interstices in the slope protection.

The forces involved in these phenomena may be greatly in excess of the forces resulting from the steady flow of a water jet against a plane surface cited in the report of the subcommittee. Furthermore, the preceding statements emphasize the error involved in attempting to apply the Ishbash (Ishbach) studies[5] to the grossly different phenomena of wave action on a structure.

Although, for a long time, designers of shore protection and harbor structures have recognized the phenomena previously described, satisfactory procedures are not yet available to compute the effective forces. The subcommittee reports correctly the need for research in this field. It is hoped that the large wave tank being designed for construction at the Beach Erosion Board will be a powerful and exceedingly useful tool in such research. However, need for a skilfully prepared and adequately supported program of research designed to utilize this facility as effectively as possible must be recognized.

The writer ventures to suggest, as a worthy project for the subcommittee, the development of a research plan that would be designed to supply ultimately a thorough understanding of the problems in earth dam upstream slope protection as well as to provide a sound basis for the design of protective works. The contribution made by the subcommittee in its present report merits the continued support of the profession in its efforts to establish reliable solutions to this important problem.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852

DISCUSSIONS

VALUATION AND DEPRECIATION OF PUBLIC UTILITY PROPERTY

Discussion

By J. Kappeyne

J. Kappeyne,[25] M. ASCE.—During the half century that has elapsed between the decision of the United States Supreme Court in the Smyth versus Ames case (169 US 466) and its decision in the Hope Natural Gas Company case (320 US 591), the economic and legal status of public utilities has undergone considerable change.

In the latter part of the nineteenth century public utilities were competing with private enterprises, having all the attributes of unrestrictive private property. Today public utilities are regulated monoplies from which the right to speculative gains inherent in unrestrictive private property has been taken. Regulation and speculation are inconsistent, regulation being merely a substitute for competition.

It may well be that the Supreme Court had this change in legal status in mind when it made its decisions in the Natural Gas companies cases. In these cases the Court merely held that, if the rate fixed is just and reasonable, it is not compulsory to consider reproduction cost in arriving at the end result. It would seem that, if this were given some further consideration, the "confusion worse confounded" may be somewhat clarified.

As the writer [26] has stated previously, it is unfortunate that several uniform systems of accounts in effect today include the terms "value" and "depreciation." The meaning of these two words is defined in the classification itself and is different from their usual meaning in the economic sense. The fact that this is so, however, makes it pertinent for the engineering profession to make clear what meaning is intended in each given instance and not to confuse conclusions based upon different meanings.

Note.—This paper by Maurice R. Scharff was published in June, 1948, *Proceedings*.

[25] Cons. Engr., Brooklyn, N. Y.

[26] *Proceedings*, ASCE, September, 1944, p. 1171.

In the economic sense "service value" means the value to the consumer, of the service rendered; "depreciation" means a loss in value, and not a loss in service capacity resulting in the ultimate retirement of property. Both terms relate to value.

In these uniform systems of accounts these terms relate to cost and not to value. "Service value" is the difference between original cost and the net amount received for property retired, less the cost of removal. "Depreciation" is the loss in "service value" (as defined herein) incurred in connection with the prospective retirement of the plant. Merely to recite these accounting definitions shows their artificiality and the confusion of tongues that their use can and has induced.

Apparently Mr. Scharff has in mind the economic meaning of the terms "value" and "depreciation." However, when he recommends using a certain ratio relating to value as a measure of the "depreciation reserve requirement" (which is an accounting concept), that confusion previously mentioned as occurring between economic and accounting concepts seems to be present.

The theory of determining future rates based on the cost of rendering service by a substitute plant of the most modern and efficient kind available is interesting from an economic viewpoint. However, it is feared that such a process is too complicated to be applied by regulatory authorities in that it is solely dependent upon estimates, which undoubtedly will vary with the views as submitted by various experts, not only as to applicable costs, but also as to basic design. Such complications may be far worse than those attendant upon the fixing of the old-fashioned "reproduction cost," and thus are of little value to the fundamental problem to be answered by the regulatory authority.

The owners of the utility—that is, the investors who furnished the required capital—not only are entitled to the protection of their investment, but also are entitled to the opportunity of earning a return thereon that they had good reason to expect at the time the investment was made. Once an investment is made in a monopolistic, public utility enterprise, the investors are compelled to keep the facilities extended in advance of those needed to serve their customers adequately.

The present accounting methods, prescribed by the regulatory authorities, permit the determination, by years, of the cost of the surviving facilities, including elements of working capital. The historical rates of money for such years may then be weighted, resulting in an average rate of return that the investors had reason to expect when they had to make their investments. One would reach substantially the same end by multiplying the present average rate, at which money for utility investments can be obtained, by the weighted average purchasing power of the dollar over the years, in which the investment was actually made. This would seem to be the easier and more practical solution for determining the rate of return which is equitable to both the investors and the customers, subject to possible adjustments as warranted by circumstances in the particular instance.

The more consistent and simple the regulatory method of determining the rate base, the better able will the utility be to prepare for the future. The fluctuation in the value of the dollar, the realized return on funds invested that do not take the full risk of the enterprise, the efficiency of management, and the other economic factors that will require special consideration may then be compensated for in the rate of return allowed rather than by a revised method of determining the rate base. If these factors are not taken into consideration, an insufficient rate of return results and thus prevents attracting new capital on favorable terms.

These comments are made for the purpose of drawing from the engineering profession further clarifying discussions of a subject which is of vital interest to the welfare of the entire nation.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

LATERAL EARTH PRESSURES ON FLEXIBLE RETAINING WALLS A SYMPOSIUM

Discussion

BY D. P. KRYNINE

D. P. KRYNINE,[60] M. ASCE.—All papers of the Symposium are interesting and merit attention. New ideas are advanced and considerable material for further thinking and research is furnished. The sponsorship of the United States Navy Department is to be highly commended. Skill, energy, and persistence in work by individual authors, particularly by Professor Tschebotarioff, must be given due credit.

Large-Scale Model Earth Pressure Tests on Flexible Bulkheads.—The lateral pressure on a perfectly immovable, indeformable, and frictionless wall obviously would be the same as the horizontal pressure within a natural earth mass made of the same material as the backfill. The lateral pressure diagram in the case of a rigid wall with a backfill possessing some shearing strength may change its triangular shape as the result of translation (yield) of the wall. In the case of a flexible wall, such as a bulkhead, such a change is due to: (*a*) Translation of the wall because of the stretching of the tie rod and forward motion of the anchor and the foot of the sheet piling, and (*b*) deflection of the sheet piling. In the case of a wall that is assumed to be flexible, but in reality is not flexible enough, and in addition is restricted in its forward movement, the lateral pressure diagram would be approximately of a triangular shape. Fig. 18(*b*) indicates that this possibly was the case of the model used in the tests at Princeton University (Princeton, N. J.). This diagram is of great value as a general guide for the designer, however. In fact, a bulkhead will yield at first, being pushed forward by the pressure of the practically liquid, hydraulically placed backfill. There is no redistribution of pressures at this initial stage since liquids do not possess shearing strength great enough to permit shearing stresses of appreciable magnitude to be developed in the backfill. This is the most dangerous time in the service of a bulkhead; and, therefore, the structure should be designed for a triangular lateral pressure distribution.

NOTE.—This Symposium was published in January, 1948, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: June, 1948, by Ralph B. Peck, Howard C. Roberts, Armand Mayer, S. Packshaw and J. Owen Lake, and Jacob Feld; and September, 1948, by E. De Beer.

[60] Dept. of Civ. Eng., Yale Univ., New Haven, Conn.

Required Value of Yield.—The proper way to state the value of the yield of the wall that may produce changes in the lateral pressure distribution is to express it in absolute values (for instance, in inches), rather than in fractions of the span of the wall as usually done. The larger the tendency to the relative displacement of one horizontal soil layer with respect to the other, the larger will be the reaction opposing this movement. A large wall may be flexible, whereas its model may be rigid. In particular, the maximum deflections of the testing plate shown in Fig. 17 are between 0.4 in. and 0.5 in.; and, since the pressure distribution was practically triangular (see Fig. 18(*b*)), it should be concluded that the absolute value of these deflections was insufficient to produce a lateral pressure redistribution. In the prototype, however, the same deflections, duly increased as shown hereafter, would possibly be sufficient to produce lateral pressure redistribution.

Similitude Problem.—The model used in the tests was about one fifth of the prototype size. Increasing the model five times to the size of the prototype, the thickness of the wall would be 5 in. The web thickness of common, full-size, sheet pile sections[61] is only from $\frac{3}{8}$ in. to $\frac{1}{2}$ in. The section modulus of the pile type M107 is 3.0 in.^{3} per ft and that of the heaviest type (M110) is 15.3 in.^{3} per ft, whereas a plate 5 in. thick would have a section modulus of 50 in.^{3} per ft. When computing deflections in engineering textbooks and handbooks the physical properties of the load are not considered at all, the load being shown schematically by arrows. In such a case a fivefold increase of the model would cause a fivefold increase of unit lateral pressures. The moments of inertia (per foot of width) would increase 125 times and the deflections would increase twenty-five times. This would create yields of the prototype that may reach $0.4 \times 25 = 10$ in. However, because of the interaction of elastic forces in the bulkhead and in the material of the fill, such an estimate is exaggerated. Assuming more conservatively that the yield of the wall in the prototype would be between five times and twenty-five times larger than that in the model, there may be yield values of 5 in, or 6 in. that are already sufficient to mobilize shearing stresses of appreciable magnitude in the backfill and thus possible modify the pattern of lateral pressure distribution shown in Fig. 18(*b*).

Decrease of Lateral Pressure Due to Consolidation.—A sheet pile, deformed by hydrostatic pressure of the hydraulic fill, tends to return to its original shape and size and does so as soon as consolidation begins. As the consolidation process continues, vertical and horizontal shears are created in the backfills. The value of these horizontal shears is increased by the elastic action of the material of the bulkhead since the latter decreases its curvature and presses on the backfill. In this sense the lateral pressure as shown in Fig. 17 should be identified as passive pressure rather than as active pressure (stages 4 and 10). The passive pressure below the tie rod acts from the sheet piling toward the backfill, and the displacements are in the same direction. Possibly this will clarify Professor Tschebotarioff's doubt as to the validity of conventional stress-strain equations in the case of consolidation (Fig. 17 and supporting text).

[61] "Carnegie Pocket Companion," Carnegie-Illinois Steel Corp., Pittsburgh, Pa., 1938.

Tension Zone at the Top of a Cohesive Earth Mass.—The tension problem is of importance in the case of unsupported slopes, or vertical slopes supported by yielding rigid retaining walls, rather than in the case of bulkheads. For the case of rigid walls, the writer shares Professor Tschebotarioff's opinion as to the nonvalidity of the formula:

$$z_0 = 2\frac{c}{\gamma} \qquad (64)$$

for determining the depth of the tension zone at the horizontal top surface of a cohesive earth mass (Fig. 18 (*a*)). Eq. 64 may be valid for a stretched semi-infinite mass (if only such a mass can be stretched because of the lack of space required for displacements), but it has no significance in the case of a finite vertical slope, unsupported or supported by a yielding retaining wall. In fact, Eq. 64 may be developed considering the Mohr circle for a point with zero tension located at a depth z_0 below the top of the slope.[12] In the case of a stretched semi-infinite mass, the principal stresses at the given point are practically the vertical pressure $\gamma\, z_0$ and the zero tension. In the case of a finite mass, however, the vertical pressures at the points located at a horizontal plane z_0 units deep are not constant, but vary from point to point. The presence of a vertical slope affects the stress distribution in the mass materially only on a short distance from the slope, this distance being perhaps equal to about the height of the slope. Consider conditions of equilibrium of the prism ABCD, Fig. 67, keeping in mind the condition $\Sigma M \doteq 0$. The moment of the couple formed by the total pressure at rest F above plane BD (depth z_0) and the corresponding reaction F equals $\frac{1}{3} F\, z_0$. To balance this couple another couple I–II, Fig. 67, is necessary. Estimating very roughly the arm of this couple at $z_0/3$, and remembering that

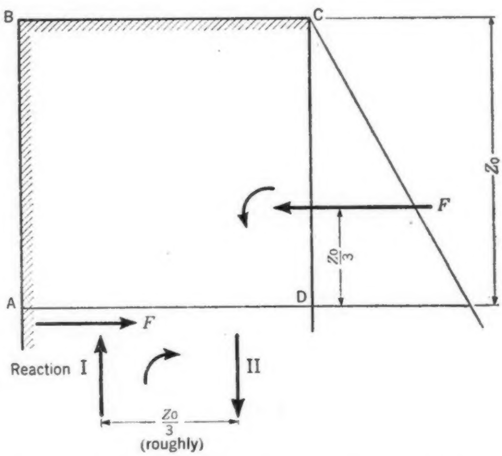


FIG. 67.—CONDITIONED $\Sigma M = 0$ FOR THE END ABCD OF AN UNSUPPORTED EARTH MASS

$$F = \tfrac{1}{2} K\, \gamma\, z^2_0 \qquad (65)$$

(in which K is the coefficient of pressure at rest) the value of each of the forces I and II (Fig. 67) would be F. Distributing the overloading force F uniformly over an area of about $z_0/3$, the average additional pressure at a point of plane BD next to the slope is

$$\frac{0.5\, K\, \gamma\, z^2_0}{z_0/3} = \frac{3}{2} K\, \gamma\, z_0 \qquad (66)$$

[12] "Theoretical Soil Mechanics," by Karl Terzaghi, John Wiley & Sons, N. Y., 1943, p. 96.

For $K = 0.6$, the average additional pressure at a point in plane BD next to the slope may reach a value of $0.9 \gamma z_0$. In reality the overloading and relieving stresses at different points of plane BD are not uniformly distributed, as already stated. More information along these lines has been given by the writer elsewhere.[62]

If a retaining wall is perfectly immovable, there is no overloading of the backfill at the surface of contact with the retaining wall. Any translation of the latter (and hence any decrease in the value of the reaction of the wall) approaches a stress pattern similar to that shown in Fig. 67.

Application of Test Results to Quay Wall Design.—Mr. Epstein discusses the possibility of decreasing the active pressure on a bulkhead from a hydraulic backfill by placing a sand dike next to the latter (Fig. 38) or by using vertical sand blankets. Professor Tschebotarioff's interesting tests along these lines have shown that blankets narrower than one tenth of the height of the bulkhead are not efficient. In so far as the dikes are concerned, the writer believes that the difference between the pressure from the hydraulic fill and that from the sand dike is taken up by the earth mass below the bulkhead. In other words, the dike does not decrease the pressure from the hydraulic backfill, but simply redistributes it. To claim otherwise is to contradict the condition of equilibrium $\Sigma H = 0$. It may be argued that pressure can be transmitted to an earth mass or a part of an earth mass, only through some physical body different from the earth mass itself, such as a masonry base or a concrete wall. This is not so, however. As an example, consider the increase in lateral pressure above an unlined tunnel. At a certain distance from the tunnel the natural lateral pressure follows a triangular pattern. This pressure must be balanced at any vertical plane of the mass including the center line of the tunnel; and in this case the condition $\Sigma H = 0$ is satisfied by the increase of the natural earth pressure (that is, the pressure existing prior to the construction) above and possibly below the tunnel.

The shearing stress diagram below and within the dike or a blanket must have a certain shape and a certain maximum value that does not exceed the shearing strength of the material of the dike (or the material at the base of the dike). To be developed, such a curve[62] requires a certain length; and apparently the length of one tenth of the height of the wall is not sufficient for this purpose.

Failure of Quay Wall at Mare Island, California.—Commander Coxe is to be commended for a lucid presentation of the case including the description of the sand drain method of O. J. Porter, M. ASCE. The Mare Island failure was really an object lesson for designers and for the civil engineering profession as a whole. It was proof of the view that, if the submerged backfill consists of a material with a flexible skeleton, the total value of the surcharge is transmitted to the wall by the pore water. It is obvious that, since the lateral pressure on a bulkhead decreases as consolidation progresses, the most dangerous period in the life of a bulkhead is the time of its construction and perhaps a certain brief time afterward.

[62] "Vertical and Horizontal Shearing Stresses in Earth Masses," *Proceedings*, 2d International Conference on Soil Mechanics and Foundation Eng., Rotterdam, Holland, 1948, Vol. VII, Paper No. Ie22.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

HIGHWAY BRIDGE FLOORS A SYMPOSIUM

Discussion

BY E. W. WENDELL, AND H. TACHAU

E. W. WENDELL,[27] M. ASCE.—A coordinated, forward step in the matter of determining the theory to be applied in the design of composite bridges is represented by this Symposium. In the writer's opinion, results obtained in this manner, from actual tests, have a far better chance for acceptance by practicing engineers than does any purely theoretical approach. Regardless of any theory which can be utilized as a vehicle of approach to a problem, most working engineers react favorably to model tests and full-size tests when carried out under the type of supervision extended by the authors.

Interest in the design of composite beam and girder bridges has been quickened for various reasons. The chief reason for this interest is that this type of structure provides a satisfactory answer to the problem of combining economy, simplicity, and beauty. In the matter of economy, however, it is well not to become overly enthusiastic about the savings in weight of the composite design as compared with the noncomposite design. A saving in weight alone in this day of high labor costs does not necessarily mean a saving in dollars. At present there must be at least a saving in weight of 20% in order to compensate for the additional costs of the spiral type of shear connectors which have been generally adopted for use, but in which there is not sufficient interest to exclude other types.

Mr. Siess indicates that from his investigation of welded steel beams a limiting web thickness of not less than one eighty seventh of the clear depth is desirable. This is stated to provide adequate safety against buckling and to obviate the need for intermediate web stiffeners. Specifications[18] of the American Association of State Highway Officials (AASHO) require that the web thickness shall be not less than one sixtieth of the clear depth before intermediate stiffeners can be eliminated. Picking at random a structure from the design table and assuming that one eighty seventh of clear depth would be satisfactory,

NOTE.—This Symposium was published in March, 1948, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: September, 1948, by Searcy B. Slack.

[27] Deputy Chf. Engr., State Dept. of Public Works, Albany, N. Y.

[18] "Standard Specifications for Highway Bridges," AASHO, Washington, D. C., 4th Ed., 1944.

the case of a 115-ft all-welded beam may be investigated The web plate would be 57 in. by $\frac{5}{8}$ in. and would weigh 14,000 lb for one 115-ft span. By careful analysis, it is possible to use a web plate 57 in. by $\frac{3}{8}$ in., with a total of fifty-four intermediate plate stiffeners, properly spaced. The weight of this web plate plus stiffeners is 10,000 lb for one 115-ft span. This clearly shows that the use of the thinner web plate, properly stiffened, saves 28% in the weight of steel. Of course, there might well be a desire on the part of the designer to avoid the use of stiffeners for esthetic reasons, in which case economy would not be one of the controlling factors. The cost of the additional welding must also be considered, as previously mentioned in the matter of shear connectors.

It is indeed gratifying to engineers to read the conclusions which Mr. Siess has indicated with regard to the greater live-load moment taken by a properly proportioned composite bridge than by a noncomposite structure. It has frequently been suspected that the individual beams in a composite bridge take more live loads than those in a noncomposite structure, and Mr. Siess gives an acceptable reason for this phenomenon. Certainly it is reasonable to conclude that composite beams having a greater stiffness produce less lateral distribution of superimposed loads to adjacent beams. The findings of Mr. Siess as to the value of unsymmetrical sections as compared to symmetrical sections, when utilized in composite designs, are again in conformity with what has been recognized for some time. In Table 3 there are listed the relative weights of steel beams in composite and noncomposite I-beam bridges.

The comparison clearly indicates an over-all saving of 24% for composite sections, using symmetrical rolled beams and cover plates. It would seem as if the additional savings of 7% for all-welded beams, as compared to rolled beams having cover plates, would be more than overcome by the additional welding costs. Of course, there are many instances when the depth ratio required by the design specifications for a particular span eliminates the consideration of rolled beams with cover plates. When this condition prevails, the all-welded beam must be used. Under present market conditions, it is also frequently necessary to recognize the availability of materials. It might well be that plates can be procured more readily than rolled beams. When this occurs the all-welded beam, of course, should be specified. As a matter of fact, from experience in the writer's office and as a recognition in Mr. Siess' paper, it is thought that rolled beams with attached cover plates can be specified for spans up to about 90 ft. For spans greater than 90 ft an all-welded beam should be used. When it comes to rolling unsymmetrical beams, one is entering a field from which the steel companies have retreated consistently since 1938. The variations in spans, loadings, and spacings, of beams would require too many different types of rolled, unsymmetrical sections. Steel companies are reducing the number of symmetrical sections, and unless there is a marked change in this trend the suggestion of producing unsymmetrical sections would meet with little enthusiasm.

The use of temporary supports for composite bridges does not produce the advantages that have been indicated many times in the past. It seems to the writer that the cost of providing temporary supports beneath the beams until the concrete slab has set will, in the vast majority of structures, more than

offset any possible additional savings in the weight of steel effected by having the composite section take the dead load as well as the live load. There may be exceptional cases, of course, where temporary supports can be used to effect a saving but, in observing the design of several thousand structures, the writer fails to recognize many locations where this saving could be clearly established.

The records of the results of the tests made to ascertain the slip between concrete and steel, the ensuing effect of this slip on strain in the composite section, the effective width of concrete as T-beam flange, the selection of modular ratio, and the plastic flow of the slab constitute interesting features in the paper. The tests clearly indicate that by using shear connectors the concrete and steel actually do act together as a single composite section, thereby verifying the transformed section method as a basis of design. The tests also prove that the distribution of compressive stress in concrete is substantially uniform, both at the point of maximum moment and at other sections some distances from the location of maximum moment.

Although the writer is inclined to believe that the investigators used various types of shear connectors, Mr. Siess reports on only one type, which is clearly indicated as satisfactory. The tests show that the compressive strength of concrete is important in its control of the effectiveness of shear connectors. The New York State Department of Public Works has assumed 3,000-lb concrete in its designs with satisfactory results. The AASHO specifications covering shear devices to be used in composite construction recognize the importance of both the concrete and the shear connectors in the functioning of a composite beam. These specifications in part state:

> "The shear devices shall be of such construction as will permit a thorough compaction of the concrete mass and will insure entire surfaces of shear devices being in contact with the surrounding concrete. The nature of the shear devices shall be such as to prevent a vertical separation of the slab and beam or else additional means shall be provided for this purpose."

The writer contends that various types of shear connectors can give satisfactory results. The spiral used in designs in the writer's office complies with the requirements of the AASHO specifications, and has the additional advantage that the placing of the transverse reinforcement is not difficult to accomplish. In addition the steel reinforcing bars from which the spirals are made are more easily obtained than small channels. From the practical standpoint, the ideal accomplishment would be to develop a composite type of construction which utilizes no special shear connectors. It might well be that the transference of horizontal shear between the steel beam and the concrete slab could be effected by using the transverse reinforcing bars in the slab for this purpose. By threading the lower slab bars through the beam web and by welding the top slab bars on top of the top flange, an effective composite action could be effected.

In conclusion, the writer is unable to become concerned with the plastic flow of the slab. Quite naturally, $n = 30$ is used for added dead loads because they are required in the specifications. This is an unnecessary requirement; the small additional load of the wearing surface, sidewalks, and railings could be provided for with the usual value of $n = 10$ rather than $n = 30$.

It would be well to analyze carefully the "methods of proportioning composite beams," as indicated through the use of empirical formulas. To an experienced designer, the cut-and-try method of ascertaining the proportions of a composite beam is not as laborious as has been indicated. Two trials at the most, with a recognition of proper adjustments, almost always result in an efficient and economical distribution of flange areas. Experience proves that it is no more difficult to ascertain a proper distribution in this type of beam than in the case of ordinary plate girders.

Certainly engineers who are interested in composite construction should be extremely grateful to the men who have so carefully and so competently advanced the investigations. It should be, of course (and undoubtedly is), recognized that their efforts would not have been possible if it were not for those who desired the investigations and provided the necessary finances and the physical plant.

H. TACHAU,[23] JUN. ASCE.—In view of the increased activities in highway bridge construction and design, this paper is timely and especially valuable. Large sums of public funds are spent on highway bridges every year, and it is only fitting that a certain part of the engineer's efforts should go into research and development of new methods of design, such as those presented in this paper.

The purpose of this discussion is to compare the new method for proportioning beams with the 1944 specifications[18] of the American Association of State Highway Officials (AASHO) and to elaborate somewhat on the selection of the beam spacing.

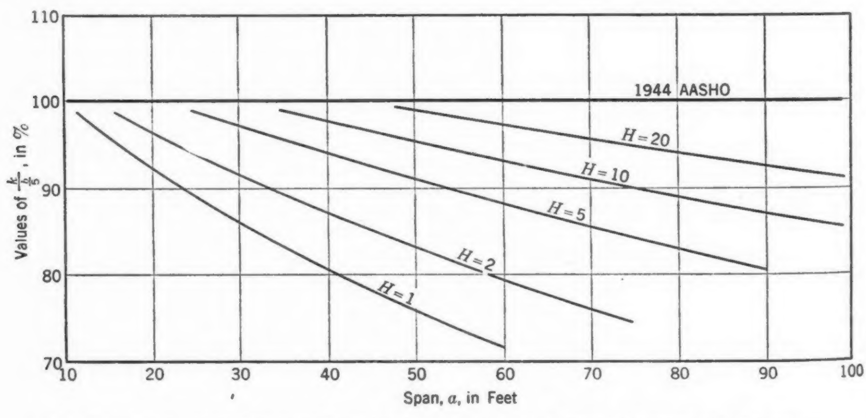


FIG. 37.—COMPARISON BETWEEN RECOMMENDED LIVE-LOAD DESIGN MOMENTS FOR BEAMS AND 1944 SPECIFICATIONS OF THE AMERICAN ASSOCIATION OF STATE HIGHWAY OFFICIALS

Fig. 37 shows at a glance how the recommended live-load design moments for the I-beams are related to values computed according to the 1944 AASHO specifications. By plotting the ratio of k to $b/5$ versus the span length a, a comparison is obtained which holds true for any beam spacing. The quantity

[23] Chicago, Ill.

$b/5$ gives the proportion of the wheel loads carried by one beam, according to AASHO specifications. In other words, it corresponds to k as used in the paper. Therefore, the AASHO value is shown as 100% in Fig. 37, regardless of the span. The recommended moments are lower, depending not only on the span, a, but also on the relative stiffness of slab and beam, H.

Inasmuch as the new method is based on extensive model tests, as well as on theoretical analyses, it appears that in some cases the AASHO method results in overdesign of the I-beams. The difference is most pronounced for small values of H and for longer spans—that is, when the stiffness of the slab is high compared to the stiffness of the beams.

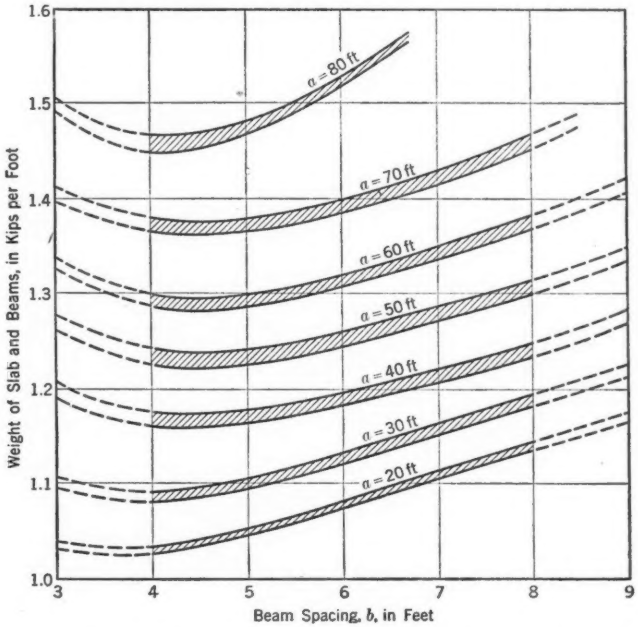


FIG. 38.—VARIATION IN WEIGHT OF SUPERSTRUCTURE (SLAB AND BEAMS FOR A 10-FT LANE) FOR VARIOUS SPANS AND BEAM SPACINGS

Very little has been written about an important phase of the structural design—the selection of the beam spacing. Except when the spacing is set by functional requirements or by architectural considerations, the designer has a wide choice. Proper selection of the beam spacing is a highly complex matter requiring good judgment. Closer spacing means more rockers, rollers, and bearing plates, and more steel will be needed for the diaphragm connections. These disadvantages are balanced by saving concrete in the deck and by using shallower beams, usually accompanied by a reduction of earthwork in the approaches. Some designers believe that the saving thus effected may, at times, offset the additional cost of steel details. Wider spacing, on the other hand, requires a heavier slab, but the cost of steel fabrication and concrete forms will be reduced.

A study of the dead loads of several designs using different beam spacings may prove helpful for making a good selection. The smallest weight per traffic lane would indicate the optimum beam spacing. Such a comparison has been made in Fig. 38.

The calculations for Fig. 38 are based on a typical cross section of the concrete slab and beams, not including sidewalks. The loading is H20-S16-44, by AASHO specifications. All beams are wide-flange beams without cover plates, designed according to the procedure presented in the paper. Composite action is not considered. For n, the ratio of the modulus of elasticity of steel to that of concrete, a value of 8 is used. The slab is of uniform thickness except over the beams, the thickness varying from a minimum of $6\frac{1}{2}$ in. for 3-ft and 4-ft spacings to $7\frac{3}{4}$ in. for 9-ft spacing, regardless of the span. These thicknesses include the wearing surface. To take account of concrete "fillets" all concrete weights are increased by 10%. The weights of the steel beams are increased by 20% to account for diaphragms, connections, splices, and other miscellaneous details.

It may be of interest to point out that usually the pavement has camber and crown; that is, it is curved in the transverse and longitudinal directions. The beams are ordinarily straight between splice points, and the space between the top of the beam and the bottom of the slab is taken up by so-called concrete "fillets," cast monolithic with the pavement slab.

Of course, the curves of Fig. 38 apply strictly only to the previously assumed values. However, they will retain their general shape if other design values are used. A change in the value of n, for instance, implies a larger H and a larger k; consequently, the curves will be shifted upward. A set of curves such as those shown in Fig. 38 may easily be constructed for any given conditions.

Possibly, it might be presumed that, for longer spans, larger beam spacings and thicker pavement slabs are in order. Nevertheless, the curves of Fig. 38 indicate that, for the assumptions stated, spacings of from 4 ft to 5 ft are best. Closer spacing seems to be preferable for short spans, also. Similar curves based on the 1944 AASHO specifications would show the same trend. As the possible saving of material appears to be less important for shorter spans, other considerations might govern. In many cases the relative cost of concrete forms and steel fabrication may furnish a relevant basis for the proper selection.